### Exercise 19  Bayesian Networks

Consider the following three-dimensional probability distribution:

<table>
<thead>
<tr>
<th>$p_{ABC}$</th>
<th>$A = a_1$</th>
<th>$A = a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B = b_1$</td>
<td>$B = b_2$</td>
<td>$B = b_1$</td>
</tr>
<tr>
<td>$C = c_1$</td>
<td>$4/24$</td>
<td>$3/24$</td>
</tr>
<tr>
<td>$C = c_2$</td>
<td>$2/24$</td>
<td>$3/24$</td>
</tr>
</tbody>
</table>

Check whether the graph depicted next to the table can be the underlying network structure describing the distribution! If yes, specify the probability distributions that are needed to define the Bayesian network!

### Exercise 20  Bayesian Networks

Consider the following four-dimensional probability distribution:

<table>
<thead>
<tr>
<th>$p_{ABCD}$</th>
<th>$A = a_1$</th>
<th>$A = a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B = b_1$</td>
<td>$B = b_2$</td>
<td>$B = b_1$</td>
</tr>
<tr>
<td>$C = c_1$</td>
<td>$D = d_1$</td>
<td>$16/82$</td>
</tr>
<tr>
<td>$D = d_2$</td>
<td>$4/82$</td>
<td>$1/82$</td>
</tr>
<tr>
<td>$C = c_2$</td>
<td>$D = d_1$</td>
<td>$4/82$</td>
</tr>
<tr>
<td>$D = d_2$</td>
<td>$4/82$</td>
<td>$4/82$</td>
</tr>
</tbody>
</table>

Check whether the graph depicted next to the table can be the underlying network structure describing the distribution! If yes, specify the probability distributions that are needed to define the Bayesian network!
Exercise 21  Probabilistic Propagation

Consider the following Bayesian network and the corresponding (conditional) probability distributions:

\[
\begin{array}{c|cc}
P(A) & a_1 & a_2 \\
\hline
b_1 & 0.4 & 0.6 \\
\end{array}
\quad
\begin{array}{c|cc}
P(B|A) & a_1 & a_2 \\
\hline
b_1 & 0.1 & 0.6 \\
b_2 & 0.9 & 0.4 \\
\end{array}
\quad
\begin{array}{c|cc}
P(C|B) & b_1 & b_2 \\
\hline
c_1 & 0.4 & 0.8 \\
c_2 & 0.6 & 0.2 \\
\end{array}
\quad
\begin{array}{c|cc}
P(D|B) & b_1 & b_2 \\
\hline
d_1 & 0.7 & 0.2 \\
d_2 & 0.3 & 0.8 \\
\end{array}
\]

a) Determine the a-priori distribution for all four variables!

b) It becomes evident that variable \( C \) assumes value \( c_2 \). Propagate this evidence across the network with the tree-based propagation algorithm presented in the lecture, i.e., compute all four a-posteriori distributions!