Exercise 22  Probabilistic Propagation

Consider the following Bayesian network and the corresponding (conditional) probability distributions:

\[
\begin{array}{|c|c|c|}
\hline
P(A) & a_1 & a_2 \\
\hline
& 0.6 & 0.4 \\
\hline
P(B|A) & a_1 & a_2 \\
& b_1 & 0.3 & 0.7 \\
& b_2 & 0.7 & 0.3 \\
\hline
P(C|B) & b_1 & b_2 \\
& c_1 & 0.6 & 0.2 \\
& c_2 & 0.4 & 0.8 \\
\hline
P(D|B) & b_1 & b_2 \\
& d_1 & 0.9 & 0.4 \\
& d_2 & 0.1 & 0.6 \\
\hline
P(E|D) & d_1 & d_2 \\
& e_1 & 0.75 & 0.5 \\
& e_2 & 0.25 & 0.5 \\
\hline
\end{array}
\]

a) Determine the a-priori distribution for all four variables!

b) It becomes evident that variable \( D \) assumes value \( d_2 \). Propagate this evidence across the network with the tree-based propagation algorithm presented in the lecture, i.e., compute all four a-posteriori distributions!

b) After some time we get additional evidence about \( A \). Assume \( a_2 \) being the observed value of \( A \) and propagate the evidence across the network with the tree-based propagation algorithm presented in the lecture. Which a-posteriori distributions are influenced by the additional evidence?
Exercise 23  Construction of Clique Trees

Construct stepwise for the depicted Bayesian network

a) the moral graph,
b) a triangulated moral graph,
c) a perfect ordering using maximum cardinality search, and
d) a cliquen tree/join tree!

At which steps of the construction do you have multiple options to proceed? Show that the resulting cliquen tree/join tree fulfills the running intersection property.

Exercise 24  Triangulation and Joint Tree Construction

Given the following three undirected graphs:

a) Check which graphs are triangulated! Try to recognize this without applying the triangulation algorithm from the lecture.
b) Triangulate the graphs that are not yet triangulated and determine for each of them a join tree!