1. Exercise Sheet

Exercise 1  Expected Payoffs

Consider a game with the following payoff matrix:

\[ A = \begin{bmatrix} C & D \\ C & [0, 0, 1, 1] \\ D & [0, 0, 1, 1] \end{bmatrix} \]

a) Compute the expected payoff for player 1 playing cooperate (C), when it is already known that player 2 plays a mixed strategy of \( x = (\frac{1}{2}, \frac{1}{2}) \).

b) Compute the expected payoff of player 1, when he plays a mixed strategy of \( x_1 = (\frac{1}{2}, \frac{1}{2}) \) and player 2 plays a mixed strategy of \( x_2 = (\frac{1}{2}, \frac{1}{2}) \).

c) Assume that both players currently play a mixed strategy of \( x = (\frac{1}{2}, \frac{1}{2}) \). What kind of strategy can player 1 play to increase his payoff? What kind of strategy can player 2 play to increase his payoff?

Exercise 2  Nash Equilibria

Consider a 2-Player Prisoner’s Dilemma game with the following payoff matrix.

\[ A = \begin{bmatrix} C & D \\ C & [3, 3, 0, 5] \\ D & [5, 0, 1, 1] \end{bmatrix} \]

What strategy profile is a Nash-Equilibrium?

Exercise 3  Nash Equilibria

Roger and Colleen play a game. Each one has a coin. They will both show a side of their coin simultaneously. If both show heads, no money will be exchanged. If Roger shows heads and Colleen shows tails then Colleen will give Roger 1 Dollar. If Roger shows tails and Colleen shows heads, then Roger will pay Colleen 1 Dollar. If both show tails, then they both get 2 Dollar.

a) Write the payoff matrix (for both players). Note: You can write in one matrix or in two matrices.

b) What is the best response of Colleen to Roger, when he plays/shows tails?

c) What is the Nash-equilibrium for this payoff matrix?
Exercise 4  Nash Equilibria in Mixed Strategies

Consider the game Rock-Paper-Scissors-Lizard-Spock. The rules are defined as:

- Scissors cuts Paper
- Paper covers Rock
- Rock crushes Lizard
- Lizard poisons Spock
- Spock smashes Scissors
- Scissors decapitates Lizard
- Lizard eats Paper
- Paper disproves Spock
- Spock vaporizes Rock
- Rock crushes Scissors

a) Write the payoff matrix for the 2-player version of the game.

b) Show that there cannot be a Nash-Equilibrium with pure strategies.

c) Proof that the Nash-Equilibrium is a mixed strategy with the probability distribution $x = (1/5, 1/5, 1/5, 1/5, 1/5)$.

- Hint: read up on the proof for the base-game Rock-Paper-Scissors and adapt it to the 5 strategy variant: [https://oyc.yale.edu/sites/default/files/mixed_strategies_handout_0_0.pdf](https://oyc.yale.edu/sites/default/files/mixed_strategies_handout_0_0.pdf)

Exercise 5  Replicator Equations and Fixed Points

Consider the Stag-Hunt game with the following payoff matrix:

$$
\begin{bmatrix}
  C & D \\
  C & [2, 2, 0, 1] \\
  D & [1, 0, 1, 1] 
\end{bmatrix}
$$

a) Use replicator equations to calculate the fixed point(s), for the number of cooperators.

b) Show to which fixed point the population converges if the initial frequency of cooperators is 75%.

c) Show which fixed point the population converges to for any start-frequency of cooperators.