Ising Model as a Switch Voting Mechanism in Collective Perception

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Abstract. This paper investigates the influence of the preferences of individuals on the process of collective perception in the collective decision-making systems. To do this, the Ising model from the context of Social Impact Theory is studied on a dynamic network of agents within an environment. This model additionally considers the mechanisms of the direct modulation of positive feedback. We propose learning rules for updating the preferences. Such rules depend on the undertaken decisions of the individuals. The experiments are evaluated on the best-of-2 collective perception problem and compared with the state-of-the-art voting mechanisms such as majority and voter models. The results show that assigning preferences to the agents allows a designer to take control over the outcome of the collective decision-making process. In addition, the agents with a right conjecture can faster reach the correct conclusion even if only 20% of the initial population holds the target opinion.

Keywords: Ising model · Collective decision making · Collective perception · Social impact theory

1 Introduction

The Ising model is a well-known model in statistical physics to study ferromagnetism, the property of a material to exhibit spontaneous aligned magnetic moments. Due to the emerging dynamics, it can be also used as a tool to study collective behaviors connected with opinion formation like consensus decision-making. The Ising model is always assumed to perform on a static network, where each agent keeps its neighbors. However, many natural collective decision-making systems (e.g. ant colonies, bird flocks) form dynamic networks, where the individuals move, change their local neighbors, and actively interact with each other and with the environment. The main goal of this paper is to investigate a variation of the Ising model as a potential voting mechanism in collective decision-making systems.

In recent years, there has been a surge of interest in the research on designing and validating collective decision-making approaches using robot swarms. Artificial systems make the study easier than dealing with natural collectives, giving a

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designer the possibility to focus on a particular decision-making phase excluding the others. Previous studies [9,11] have mainly focused on the voting mechanisms, especially the majority voting and the voter models. However, mostly one voting algorithm is assumed to be selected at the beginning by the designer and it is assigned to all the individuals, and remains unchanged throughout the whole decision-making process.

In this work, we aim to provide the individuals the capability to switch between different voting mechanisms based on their current preferences. To do this, we examine a family of nonlinear voter models with Ising-like criticality [5] and couple it with the direct modulation of positive feedback [11] on a dynamic network of agents, where each individual exerts a bias regarding a certain outcome. As the benchmark scenario, we consider collective perception [11], where a swarm of agents explores a certain environment on the availability of particular resources (features) and has to determine which is an abundant one. Although the local interactions here are different from [5], since they are modulated by internal (direct modulation) and external factors (changing interactions with others), we expect to get an adaptive collective decision-making mechanism according to [1,10]. In addition, we also study how incorporation of a learning procedure affects the decision process and enhances the model with the preference update rules, which allow the agents to change their preferences over time depending on the taken decisions.

In the remaining part of the paper, we draw the parallel between the Ising model and the social impact theory along with some corresponding background. Based on that, we then describe the considered voting mechanism together with learning update rules. Afterwards, we provide the description of the multi-agent simulation and the undertaken experiments. The results from the experiments are discussed and analyzed, and, finally, the conclusion provides a brief summary, highlighting the further research direction.

2 Related Work

2.1 Ising Model

In its original formulation, the Ising model is defined on a d-dimensional lattice, i.e. $\mathbb{Z}^d \subseteq \mathbb{R}^d$, with all coordinates as integer numbers. Let's consider the finite lattice $\Lambda_L \subset \mathbb{Z}^d$ of size L:

$$\Lambda_L = \{(i_1, i_2, ..., i_d) : |i_j| \leq L, j = 1, 2, ..., d\},$$

where i is a site in the lattice. One usually wants to work with the full infinite lattice $L \to \infty$. Each site has a discrete variable σ_i, which is called spin, and can take only two values $\sigma_i = +1$ (spin pointed "up") or $\sigma_i = -1$ (spin pointed "down"). A spin configuration σ describes an assignment of a spin value σ_i to each lattice site $i \in \Lambda_L$, i.e. $\sigma = \{\sigma_i\}_{i \in \Lambda_L}$. The total energy of the configuration σ, defines the Ising model and is calculated using its Hamiltonian:

$$H(\sigma) = -\sum_{\langle ij \rangle} w_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i, \quad (1)$$

where $\langle ij \rangle$ denotes a pair of sites in a finite Λ_L with Euclidean distance of 1: $\langle ij \rangle = \{i, j \in \Lambda_L : |i - j| = 1\}$. The parameter coefficient w_{ij} indicates the interaction between two spins σ_i and σ_j, where $|w_{ij}|$ is the strength of interaction, and h is an external magnetic field (if any). We consider the Ising model without an external field, therefore $h = 0$:

$$H(\sigma) = -\sum_{\langle ij \rangle} w_{ij}\sigma_i\sigma_j. \tag{2}$$

The probability that the system is in a state with configuration σ is called the configuration probability $\mu_L(\sigma)$:

$$\mu_L(\sigma) = exp(-\beta H(\sigma)) / \sum_{\sigma} exp(-\beta H(\sigma)), \tag{3}$$

where $\beta = 1/kT$ with the temperature T and Boltzmann constant k. The denominator is defined by the sum over all possible 2^L spin configurations on a finite lattice Λ_L. To simulate the Ising model, the single-spin-flip dynamics are established as follows: (1) In the current configuration σ, select a random spin with probability $1/L$ and flip its value (2) calculate the energy $H(\sigma')$ of a new configuration σ' (3) if $H(\sigma') < H(\sigma)$ accept the flip, else accept it only with probability $exp(-\beta(H(\sigma') - H(\sigma))$ (4) repeat the process until all the spins become aligned, i.e. the lattice becomes ferromagnetic. It is stated that the system converges in $d \geq 2$. To quantify the level of magnetization, one can calculate the average value of all the spins: $M(\sigma) = (1/L)\sum_{i=1}^{L} \sigma_i$. Although the system dynamics are studied in the thermodynamic limit ($L \to \infty$), Peierls [2] has shown that spontaneous magnetization already occurs in a relative small lattices but with the smoothed singularities due to the finite size.

2.2 Social Impact Theory

Theory of social impact is based on psychosocial laws and describes how individuals are affected by their social environment [8]. It is defined by microlevel rules (i.e. in individual level) expressed for each individual i via social force field I_i, which depends on the total number of individuals in the group N, strength of their assertiveness s_i, and either spatial or abstract (e.g. personal relations) distance between its members d_{ij}. Therefore, the social impact is considered as a multiplicative function of the described above parameters: $I = f(N \cdot s \cdot d)$.

The impact of a group with N members on the i-th individual, I_i, is calculated as follows:

$$I_i = \Big[\sum_{j=1}^{N} \frac{p_{ij}}{d_{ij}^{\alpha}}(1 - \sigma_i\sigma_j)\Big] - \Big[\sum_{j=1}^{N} \frac{s_{ij}}{d_{ij}^{\alpha}}(1 + \sigma_i\sigma_j)\Big], \tag{4}$$

$\sigma_i \in \{-1, +1\}$ is the binary opinion (e.g. yes/no) of the individual i; $\alpha \geq 0$ indicates the speed of the distance influence decline with the increase of the distance d_{ij} between the individuals. Parameter $s_{ij} \geq 0$ corresponds to the so-called supportiveness, i.e. the ability to support someone to keep the opinion and $p_{ij} \geq 0$ indicates the so-called persuasiveness, the ability to convince someone to change his opinion. Nowak and Latane [8] re-assign these parameters to some positive random values every time an individual has changed his opinion. In case, $\beta = 0$ and we have no noise in the system, the opinion σ_i changes for $I_i > 0$, and remains the same for $I_i < 0$: $\sigma_i(t+1) = -\sigma_i(t)sgn(I_i(t))$. For $\beta > 0$ (noise in the system), the probability to switch the opinion is proportional to $exp(\beta I_i(t))$.

As the result of using the above operations, a collective behavior such as polarization or fragmentation [6], can be observed on the macro-level, depending on the initial conditions. Both the social impact theory and the Ising model consider that the agents do not move in a physical space and evaluate them on a cellular automata. In comparison to statistical physics, a finite number of individuals brings difficulties into the analysis of the sociophysical models [3], since the singular behavior (i.e. order-disorder phase transition) can emerge only in the thermodynamic limit of the system ($N \to \infty$).

3 Proposed Model and Learning Process

Similar to the Ising model, we consider a swarm of N interacting individuals (instead of sites in the lattice, we take individuals), which move in a search space and explore for certain features (Fig. 1). The goal of the swarm is to collectively find the most occurring feature in the environment. In this work, we concentrate on the estimation of two environmental features, so that each individual holds its own opinion, $\sigma_i = \pm 1$, similar to the "up" and "down" spins. Different from previous approaches [11], we additionally assume that each individual has its own internal preference for a certain overall outcome, $\sigma(i) = \pm 1$. This strengthens the influence of the neighbors whose opinion correlates with own preference. In this case, the strength of the preference impact, $w_i \in \mathbb{R}^2_+$, will act similar to the supportiveness parameter in Eq. (4).

Now, we redefine Eq. (2) in a more convenient form for the z-th individual:

$$I_z = -\sum_{\langle zj \rangle} w_{zj}\sigma_z\sigma_j = \frac{1}{2}\sum_{\langle zj \rangle} w_{zj}(1 - \sigma_z\sigma_j) - \frac{1}{2}\sum_{\langle zj \rangle} w_{zj}(1 + \sigma_z\sigma_j), \qquad (5)$$

where $\langle zj \rangle$ indicates the pair of the individual z and its neighbor j. The parameter $|w_{zj}|$ is the intensity of the influence of z on j. The first term in Eq. (5) contributes if the individuals j and z have opposite opinions. The second term is non zero if the individual j holds the same opinion as the individual z and is zero otherwise. In this case, if the preference of the individual z is $\sigma_z = +1$, then it means that if there are any agents in the neighborhood with opinion $\sigma_j = +1$, the second part of the Eq. (5) has to be $|w_{zj}|$ times stronger than the first part. And vice versa, if the preference of the individual z is $\sigma_z = -1$, the first part of Eq. (5) should take an advantage over the second one. In this sense, the value of $|w_{zj}|$ represents the relation of weight between neighbors with contradiction opinion to the z^{th} individual's preference and those with the consistent one. Assuming that individual z has the same impact on all his neighbors, i.e. $w_{zj} = w_z$ for any j, we can write the following:

$$I_z = \sum_{j=1}^{\mathcal{N}_z} w_z \sigma(z)(1 + \sigma_z \sigma_j) - \sum_{j=1}^{\mathcal{N}_z} \sigma(z)(1 - \sigma_z \sigma_j) \tag{6}$$
$$= \begin{cases} w_z n_z^+ - n_z^-, & \text{if } \sigma(z) = +1 \\ n_z^+ - w_z n_z^-, & \text{if } \sigma(z) = -1 \end{cases}$$
$$= w_z' n_z^+ - n_z^-, \text{ where } w_z' = \begin{cases} w_z, & \text{if } \sigma(z) = +1 \\ \frac{1}{w_z}, & \text{if } \sigma(z) = -1, \end{cases}$$

where $\sigma(z)$ is the preference opinion of individual z, $|w_z| > 1$ is the strength of the preference and σ_z is the opinion holding by z. If $w_z' = 1$, the individual is considered as not biased, i.e. without any preference, while $w_z' \in (0, 1)$ and $w_z' > 1$ corresponds to $\sigma(z) = -1$ and $\sigma(z) = +1$ respectively. The parameters n_z^+ and n_z^- indicate the amount of the neighboring individuals with opinion $\sigma_j = +1$ and $\sigma_j = -1$ accordingly.

If we take the spatial distances between the individuals into account, we can combine Eq. (4) and Eq. (6):

$$I_z = w_z' \sum_{j \in \mathcal{N}_z^+} (1/d_{zj}^\alpha) - \sum_{j \in \mathcal{N}_z^-} (1/d_{zj}^\alpha), \tag{7}$$

where $\mathcal{N}_z^+$ and $\mathcal{N}_z^-$ correspond to the neighbors of z-th individual, which are holding opinions $\sigma_j = +1$ and $\sigma_j = -1$ respectively. In this way, the distances between z and j influence the value for I_z. Considering two individuals with different distances to z, the influence of the closer one to z is greater than the other.

The opinion dynamics are defined probabilistically using the following sigmoid function as in [5]:

$$p(\beta I_z) = \frac{1}{2}\left(1 + \frac{tanh(\beta I_z)}{tanh(\beta)}\right) \in [0, 1], \tag{8}$$

where a social field I_z is normalized, i.e. $I_z = \frac{w_z' \sum_{j \in \mathcal{N}_z^+} (1/d_{zj}^\alpha) - \sum_{j \in \mathcal{N}_z^-} (1/d_{zj}^\alpha)}{w_z' \sum_{j \in \mathcal{N}_z^+} (1/d_{zj}^\alpha) + \sum_{j \in \mathcal{N}_z^-} (1/d_{zj}^\alpha)} \in [-1, 1]$, and β acts as a noise parameter. Varying β values from 0 to ∞ allows observing a spectrum of voting mechanisms from the voter ($\beta \to 0$) to majority models ($\beta \to \infty$) and everything in between [4]. The transitions from the state $\sigma_z^t = -1$ at time t to $\sigma_z^{t+1} = +1$ at $t+1$ occur with $p(\beta I_z)$ probability, and from $\sigma_z^t = +1$ to $\sigma_z^{t+1} = -1$ with probability $1 - p(\beta I_z)$ respectively, for all swarm members.

In the following, we introduce a learning process for updating the preferences (see Algorithm 1). The main idea is based on the fact that the current opinion of the individual σ_z^t and its preference $\sigma(z)$ can be in conflict with each other (e.g. $w_z > 1$ corresponds to $\sigma(z) = +1$, while the holding opinion is $\sigma_z^t = -1$). However, when due to the interactions with the others and the environment the individual changes his opinion, so that it becomes in agreement with its preference $\sigma(z)$, we reinforce the preference strength on Δw either in a positive or in a negative side depending on the current sign of the preference (i.e. we add if $w_z > 1$ and subtract if $w_z < 1$). In case if the agent keeps its opinion in conflict with its own preference, we make a double reinforcement (i.e. $w_z \pm 2\Delta w$) in order to change the preference to the side of this opinion (e.g. (5+6) and (9+11) lines in Algorithm 1). If there is no conflict between the individual's preference and the current opinion, we keep the preference strength w_z unchangeable (lines (1+3) and (12+14) in Algorithm 1). In this way both opinions and preferences are evolved as the result of the interactions between the individuals and the environment.

Algorithm 1 Preference Update Rules

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1: if w_z > 1 & σ_z^t = +1
2:     if σ_z^{t+1} = −1, w_z = w_z − Δw;
3:     if σ_z^{t+1} = +1, w_z = w_z;
4: end
5: if w_z > 1 & σ_z^t = −1
6:     if σ_z^{t+1} = −1, w_z = w_z − 2Δw;
7:     if σ_z^{t+1} = +1, w_z = w_z + Δw;
8: end
9: if w_z < 1 & σ_z^t = +1
10:    if σ_z^{t+1} = −1, w_z = w_z − Δw;
11:    if σ_z^{t+1} = +1, w_z = w_z + 2Δw;
12: end
13: if w_z < 1 & σ_z^t = −1
14:    if σ_z^{t+1} = −1, w_z = w_z;
15:    if σ_z^{t+1} = +1, w_z = w_z + Δw;
16: end
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3.1 Multi-Agent Simulation

The environment is defined by a square grid of 20×20 cells 1×1 unit each, painted in black and white. Without loss of generality, we consider that in all environments the white color is prevailing. We consider 100 iterations in simulation as 1 s, and we plot the simulation environment every 10 iterations (i.e. 0.1 s) as it was done in previous research [9]. We use a swarm of 20 agents, initially assigned with half for opinion white ($\sigma_i = -1$) and half for black ($\sigma_i = +1$). The preferences of the individuals coincides with their initial opinions. We keep the other parameters similar to [11].

The size (diameter) of an agent is proportional to the size of the grid cells and is equal to 0.7 units (considering 1 unit = 10 cm). The agents move in the environment using a random walk executed along with collision avoidance to other agents and the borders of the grid. The random walk is performed by alternating periods of the straight linear motion and rotation on the spot for random periods of time taken from normal distribution with mean 40 s and uniform distribution between 0 and 4.5 s respectively. The linear velocity $\boldsymbol{v}$ is set to 1.6 units/s and the angular velocity ω is 7.5 rad/s. If agents collide with each other or with the borders of the grid, they randomly rotate on the spot (equally likely clockwise or counterclockwise) until their bearings will not allow them to go freely further resuming a straight motion.

Besides different movement phases, each agent i can be also in either one of the two following states going one after another: (1) *exploration* E_i, where it moves and only estimates the quality of its current opinion $\hat{\rho}_i$, or

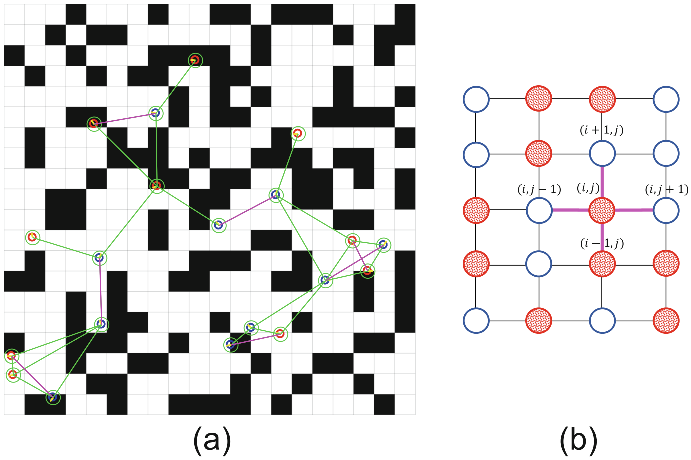

Fig. 1. Dissemination of opinions: (a) in multi-agent simulation; (b) on static lattice. Colors represent agents opinions (red for black, blue for white). Lines indicate possible connections at the current time step. Bold lines (magenta) show agents in communication with each other. The agents in (a), while moving, perceive the environment consisting of black and white cells. (Color figure online)

(2) *dissemination* D_i, when it moves and only exchanges its own opinion with the others, making at the end a decision on either to keep or to switch its current opinion i. The communication between agents is set only pair-wise, and only if both of them are in dissemination state, for each 10 iterations (0.1 s) in a random order within the communication distance of $d_{max} = 5$ units. The duration of exploration state for each individual i is the same and takes $tE = 10$ s, while the dissemination state is biased and proportional to the quality of the current opinion, i.e. $tD_i = tE * \hat{\rho}_i$, so that the less quality opinions promotes the shorter periods of time. The quality $\hat{\rho}_i$ is calculated as the ratio of time when the agent observed the color related with its current opinion i during tE. Each agent logs the opinions of its neighbors during the last 30 iterations of its dissemination state as in [11] and takes the last $\mathcal{N}$ opinions to decide based on one of the three DM strategies with $\mathcal{N} = 2$: In DMMD, the agent takes opinion which is preferred by the majority out of $\mathcal{N}$ including its own opinion. In DMVD, it adopts the opinion of a random agent from $\mathcal{N}$ excluding itself. In DC, time tD is unbiased ($tD_i = tE \ \forall i$) and at the end of tD_i each agent directly compares the quality of its own opinion i with a randomly chosen neighbor's j, and if $\hat{\rho}_j > \hat{\rho}_i$, then it switches its opinion $i \rightarrow j$ and starts E_j. The agents also transmit their individual IDs and save the received ones, so that in case, if the agent was perceiving the opinion of one and the same neighbor for two consecutive steps, only the first one is saved in the log. To validate the performance of DMs we consider two commonly used metrics: (1) *Exit probability* (E_N) to measure the ratio of successful runs among all simulations and (2) *Consensus time* ($T_N^{correct}$) as the number of iterations until all the agents converge to the correct opinion.

The run is considered successful, if the swarm has come to the consensus with the correct opinion.

4 Experimental Results and Discussion

In the first experiment, we study the impact of unbiased individuals (i.e. which do not have any intrinsic preference $w_z = 1$) on the consensus time and the exit probability for different values of the nonlinearity parameter β on the most difficult scenario, i.e $\rho_b^* = 0.92$, where ρ_b^* is the ratio of black N_{bl} and white N_{wh} cells in the grid, considering $N_{wh} > N_{bl}$: $\rho_b^* = \frac{N_{bl}}{N_{wh}}$. We examine three decision-making strategies (denoted further as DMs): Static, Adaptive, and W-Adaptive. In W-Adaptive the initial distribution of the preferences is set to the range of $(0, 1)$ for all agents despite their current opinions, while in Adaptive the initial preferences of the individuals coincide with their initial opinions (i.e. half with preferences for white and half for black). In all the experiments below, we set $\Delta w = 0.1$ in Algorithm 1 and $\alpha = 0$ in the Eq. (7), unless otherwise stated.

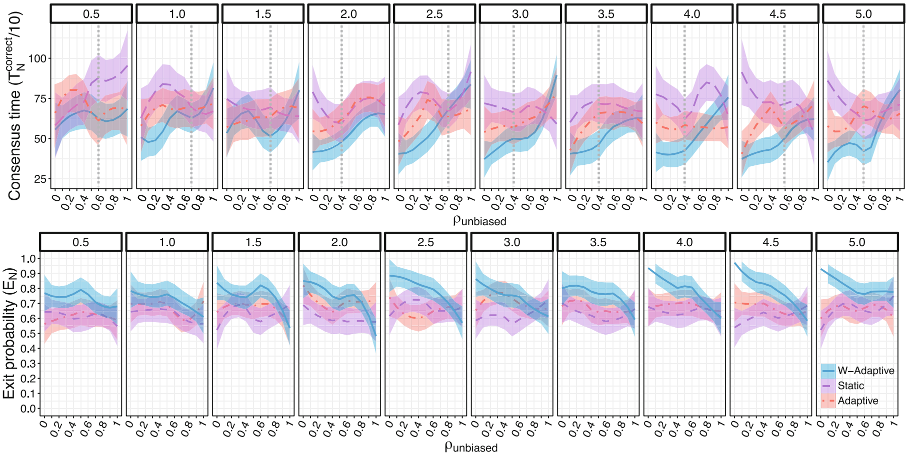

Fig. 2. Consensus time ($T_N^{correct}$) and exit probability (E_N) as a function of the proportion of unbiased individuals in the swarm (i.e. $\{0, 0.1, ..., 1.0\}$) for each value of the noise level $\beta \in \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$ (top headings). Task difficulty: $\rho_b^* = 0.92$. Dashed (pink), solid (blue) and dotdashed (red) lines correspond to the Static, W-Adaptive and Adaptive strategies respectively. (Color figure online)

Figure 2 shows the estimated smoothed conditional means of the consensus time calculated only among successful runs and the exit probability with shading areas of 95% confidence interval for the described above experiment. For all noise levels, there is a clear trend of the performance deterioration in all of the considered DMs with the increase of unbiased individuals in the population. The best performance in terms of consensus time as well as in the accuracy is mostly

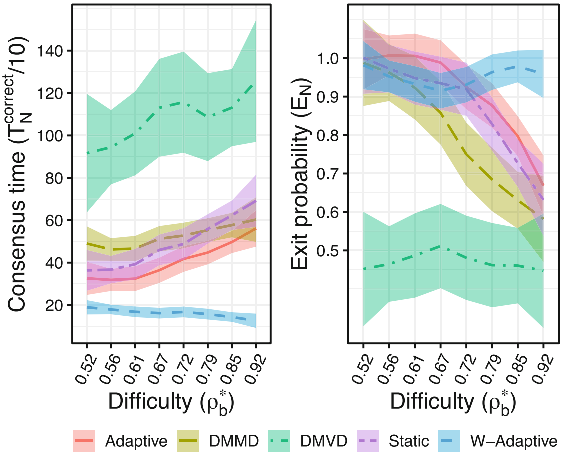

Fig. 3. Consensus time ($T_N^{correct}$) and exit probability (E_N) as a function of the task difficulty ρ_b^*. Parameter configurations: $\rho_{un} = 0$, $\beta = 4.5$. Solid (red), longdashed (khaki), dotdashed (green), twodashed (pink) and dashed (blue) lines correspond to the Adaptive, DMMD, DMVD, Static and W-Adaptive strategies respectively. (Color figure online)

at $\rho_{un} = 0$ for both adaptive strategies with $\beta \in (2.0, 5.0)$. The performance of W-Adaptive suggests that when the whole population is initially biased to the white (i.e. $\rho_{un} = 0$), the agents are able to gain a momentum and to keep their weights in $(0, 1)$ range during the adaptation process, thereby leading the collective to the fastest and the most accurate (almost 100% success rate) collective decisions among the others (see Fig. 2). However, with the introduction of unbiased individuals into W-Adaptive population, the accuracy starts decreasing at any values of β, while for Static and adaptive strategies it keeps almost stable at around $(0.65, 0.7)$ exit probability. For the consensus time, at $\beta = 4.0$, we observe a stable rate for both W-Adaptive and Adaptive until $\rho_{un} = 0.4$ along with a significant drop for Static. At other noise levels, the consensus time for W-Adaptive and Adaptive mostly increases with the increase of unbiased individuals along with a few non-significant drops at the lower values of noise, i.e. $\beta \in (0.5, 1.5)$ at $\rho_{un} = 0.6$. While for the Static, the presence of unbiased individuals promotes a significant decrease in the consensus time without compromising the accuracy for most levels of β, i.e. at $\rho_{un} = 0.4$ and $\rho_{un} = 0.5$ for $\beta = \{2.0, 4.0\}$ and $\beta = \{4.5, 5.0\}$ respectively.

The obtained results for the Static are in agreement with findings in [5], which showed that the individuals without a preference (unbiased) help to minimize the time to achieve the consensus but on the 2D square lattice. In our case with Static strategy, this finding goes further in its understanding as our results indicate that an unbiased sub-population aids to propagate exactly the *correct opinions* (i.e. with the highest quality) in the dynamic network coupled with the environment within the concept of a direct modulation mechanism of positive feedback [11]. Although, this differs for both of the considered adaptive strategies, where the unbiased agents mostly slow down the adaptation process, especially in the case of W-Adaptive. Such a result can be attributed to the fact that the weights of unbiased individuals, $w_z = 1$, keep stable for all period of time without taking part in adaptation, thereby diluting the reforming process of the already existence preferences, that leads to the increase of the consensus time. The difference in the performance between the considered strategies in Fig. 2 provide the evidence to support the above conjecture.

Additionally, we also perform the experiments of the discussed above DMs on all eight types of the task difficulty but without any unbiased individuals to examine the influence of the preferences in comparison to the two well-studied DMs in [11], namely the majority rule (DMMD) and the voter model (DMVD). As in previous studies of [11], the results in Fig. 3 indicate that the consensus time increases with the increase of the problem difficulty and the DMMD is much faster than DMVD. For most of the ρ_b^*, the Adaptive is significantly faster than DMMD but evens out with it at higher $\rho_b^* = 0.92$. The W-Adaptive is the fastest among the others with a slight acceleration in the consensus time after $\rho_b^* = 0.72$. It is approximately three times faster than the DMMD for all ρ_b^*. The Static shows the intermediate performance between the DMMD and the Adaptive with an increase in consensus time after $\rho_b^* = 0.72$. It is significantly worse than Adaptive for higher ρ_b^* (i.e. after $\rho_b^* = 0.72$). For the exit probability, the difficulty of $\rho_b^* = 0.72$ is a breaking point for the preference-based DMs. That is, until $\rho_b^* = 0.72$, Static and W-Adaptive indicate the similar accuracy trend in $(0.9, 1.0)$ range along with almost 1.0 stable rate until $\rho_b^* = 0.67$ for Adaptive. After $\rho_b^* = 0.72$, the performance of Static and Adaptive is decreasing until $(0.6, 0.7)$ exit probability, while W-Adaptive keeps the highest accuracy (i.e. $0.9 - 1.0$) also for higher ρ_b^*. The accuracy of the DMMD is lower than for the preference-based DMs starting from $\rho_b^* = 0.67$ and decreasing until the chance level at higher ρ_b^*. While the exit probability of the preference-based DMs is always higher than by the chance, the DMVD accuracy is almost always in $(0.5, 0.6)$ range despite the problem difficulty.

Altogether, these results suggest that the Ising model performs mostly similar to the majority rule in the context of the direct modulation of the positive feedback. However, the existence of preferences allows taking control over the decision-making process by manipulating the influence of neighbors whose opinions are in agreement with an individual's preference (as it is done in W-Adaptive and Adaptive).

4.1 Influence of Initial Preferences

In the following, we study the performance of the preference-based DMs *without unbiased individuals* depending on the initial number of the agents $E_a(0)$ favoring the incorrect opinion a (black), and compare it with the DMMD and the DMVD strategies. Taking into account that the total number of agents $N = 20$, we varied $E_a(0)$ from 1 to 19 with all the values in between and performed 40 simulation runs for each configuration. Figure 4-left shows that in a simple scenario $\rho_b^* =$

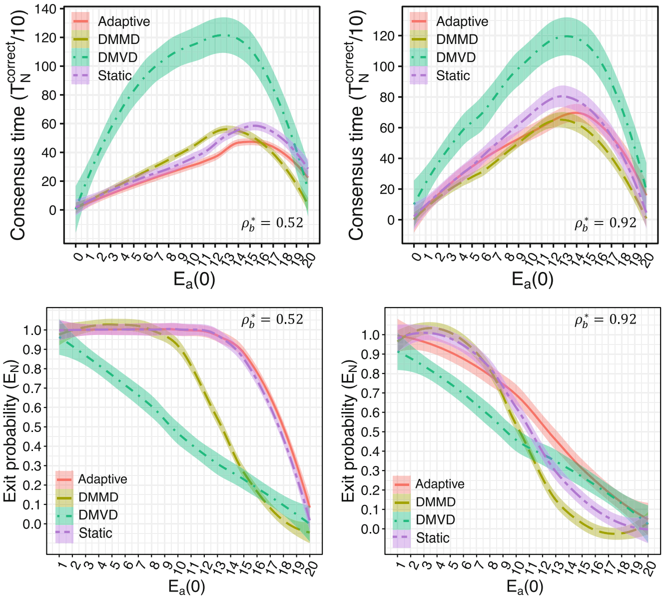

Fig. 4. Consensus time ($T_N^{correct}$) and exit probability (E_N) as a function of the initial number $E_a(0)$ of individuals with opinion a (black) in the swarm (while prevailing color in the environment is b, i.e. white). The values of initial preference bias of an individual are in accordance with its initial opinion. Parameter configurations: $\rho_b^* \in \{0.52, 0.92\}$, $\beta = 4.5$, $\rho_{un} = 0$, $N = 20$. Solid (red), longdashed (khaki), dotdashed (green) and twodashed (pink) lines correspond to the Adaptive, DMMD, DMVD and Static strategies respectively. (Color figure online)

0.52, Adaptive is the fastest strategy for all initial conditions $E_a(0)$ than the others, along with the highest accuracy holding $(0.9, 1.0)$ exit probability even with 80% of the initial population targeting the wrong opinion (i.e $E_a(0) = 16$), while the DMMD starts already degrading from $E_a(0) = 9$ (i.e. 45%). For the difficult scenario, in Fig. 4-right, the DMMD and Adaptive have similar consensus speed followed by the Static and the DMVD. Although, the decision accuracy of the Adaptive strategy decreases more slowly than the DMMD with increase of $E_a(0)$.

4.2 Matter of Distance Dependency

We have also performed the experiments incorporating the influence of the moments α of the *spatial* Euclidean distance d_{ij} between the individuals (see Eq. 7) with $\alpha \in \{-1, 0, 1, 2\}$. The obtained results (not reported here) indicate no significant impact of the *spatial distance* dependencies on the overall decision-making process, independently on the parameter α. This can be explained by the fact that agents communicate with each other in a very limited radius of max 5 units, which resulted in a small numbers not affecting the outcome of Eq. (8).

However, more experiments with unlimited interactions, i.e. global communication, are needed to support this claim.

5 Summary and Conclusion

In this work, we provided an Ising-based approach for the collective decision-making systems incorporating the dynamic preferences of the individuals. We proposed a learning procedure for the preferences, so that opinions and the preferences can co-evolve together to allow a better decision process. The Ising model can be considered as a general voting approach, integrating in itself the variety of the mechanisms from voter to majority models, which can be controlled by its non-linearity (noise) parameter. Our results indicate that the preferences give an external observer (a designer) an opportunity to manipulate the undertaken decisions of the individuals with an initial conjecture of the suggested outcome. With the right conjecture, the latter can significantly increase the speed and accuracy of the collective decision-making process even with 80% of the initial population holding the opposite opinion. The same is observed with the equally distributed preferences over the population, i.e. 50%–50% for two opinions. As part of the future work, we are going to investigate the generalization of the Ising model, which is called the Potts model [7], to explore the case of the best-of-n problem with $n > 2$.

References

1. Berdahl, A., Torney, C.J., Ioannou, C.C., Faria, J.J., Couzin, I.D.: Emergent sensing of complex environments by mobile animal groups. Science **339**, 574–576 (2013)
2. Bonati, C.: The Peierls argument for higher dimensional Ising models. Eur. J. Phys. **35**(3), 035002 (2014)
3. Castellano, C., Fortunato, S., Loreto, V.: Statistical physics of social dynamics. Rev. Mod. Phys. **81**, 591–646 (2009)
4. Drouffe, J.M., Godrèche, C.: Phase ordering and persistence in a class of stochastic processes interpolating between the Ising and voter models. J. Phys. A: Math. Gen. **32**(2), 249–261 (1999)
5. Hartnett, A.T., Schertzer, E., Levin, S.A., Couzin, I.D.: Heterogeneous preference and local nonlinearity in consensus decision making. Phys. Rev. Lett. **116**, 038701 (2016)
6. Holyst, J.A., Kacperski, K., Schweitzer, F.: Social impact models of opinion dynamics. Annual Rev. Comput. Phys. **IX**, 253–272 (2001)
7. Martin, P.: Potts Models and Related Problems in Statistical Mechanics. Series on Advances in Statistical Mechanics. World Scientific, Singapore (1991)
8. Nowak, A., Latane, B., Szamrej, J.: From private attitude to public opinion: a dynamic theory of social impact. Psychol. Rev. **97**(3), 362–376 (1990)
9. Strobel, V., Castelló Ferrer, E., Dorigo, M.: Managing byzantine robots via blockchain technology in a swarm robotics collective decision making scenario. In: Proceedings of the 17th International Conference on AAMAS, pp. 541–549 (2018)

10. Torney, C., Neufeld, Z., Couzin, I.D.: Context-dependent interaction leads to emergent search behavior in social aggregates. Proc. Natl. Acad. Sci. **106**(52), 22055–22060 (2009)
11. Valentini, G.: Achieving Consensus in Robot Swarms. SCI, vol. 706. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-53609-5