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**A Deep Learning based Approach
for Automotive Spare Part
Demand Forecasting**



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A Deep Learning based Approach for Automotive Spare Part Demand Forecasting

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List of Acronyms

ADI	Average Demand Interval
ANN	Artificial Neural Network
AR	Autoregressive
ARMA	Autoregressive–Moving-Average
BPTT	Backpropagation Through Time
CC	Correlation Coefficient
CEC	Constant Error Carousel
DE	Differential Evolution
EDO	End of Delivery Obligation
EOL	End of Life
EOP	End of Production
EOS	End of Service
GPM	Grey Prediction Model
LSTM	Long Short Term Memory
MA	Moving Average
MLP	Multi Layer Perceptron
MSE	Mean Squared Error

OEM Original Equipment Manufacturer

POC Proof of Concept

RBF Radial Basis Function

ReLU Rectified Linear Unit

RNN Recurrent Neural Network

RMSE Root Mean Squared Error

SBA Syntetos-Boylan Approximation

STPM Short-Term Prediction Model

SES Simple Exponential Smoothing

SOP Start of Production

SGD Stochastic Gradient Descent

SVM Support Vector Machine

SVR Support Vector Regression

VPD Vehicle Production Data

1. Introduction

A modern car is composed of round about 30,000 parts [93]. Components that bust over time, need to be replaced during the maintenance process. Therefore, spare parts are needed at the right place, in the right quality and quantity, for replacement of broken parts to keep the car working. As of Biedermann [7], this is controlled by spare part management. Due to the steadily increasing complexity of the products of the automotive industry over the last decades the spare part management activities also gained intricacy. This enlarged the economic importance of the spare part sector for the automotive companies. The after sales services, including all activities following the sale of the car, are holding a profit share nearly ten times larger than the car sales [86]. The spare part business generates around 50% to 70% of this revenue. According to a study of McKinsey & Company [70], this market will further grow in the next decade. This expanding market will also increase the importance of the spare part management.

The steadily growing revenues and the increasing complexity of the spare part management raise the need of optimization. This works focus is set on the optimization of the spare part demands of a worldwide operating automotive company by using computational intelligence techniques to predict future demands, based on the available historic demand data, minimizing over- or underestimations of the real demand. According to Klug [57], future spare part demands should be predicted as accurate as possible to optimize the spare part management related costs, like production, storage and transport expense, to gain a competitive advantage and raise earnings of the spare part sector.

Since 2014 IBM developed plenty of models for long-term spare part demand prediction with several worldwide operating car manufacturers. In contrast to short-term predictions these models forecast the spare part demand for a much longer time span, than the period of historic demand available for model training. Each of these models uses the historic demand data of a particular class of spare parts, to apply the approche's characteristic strengths, aligned

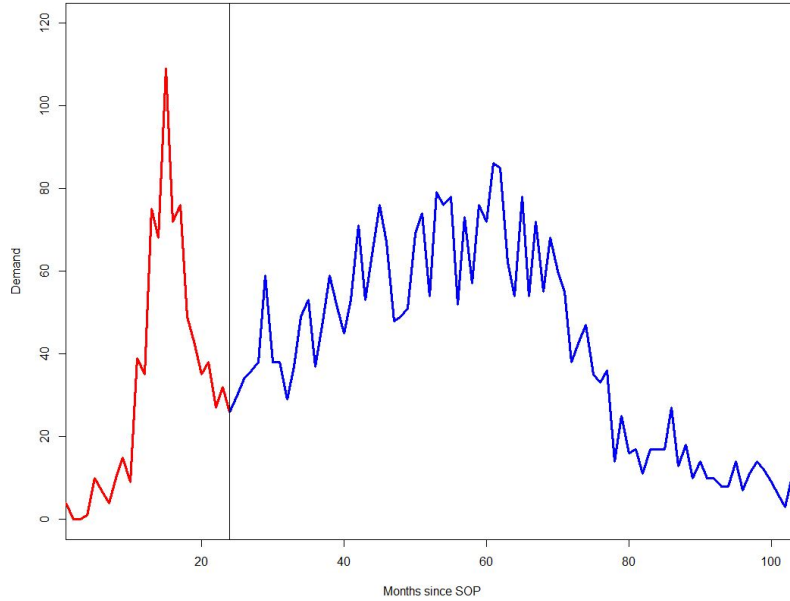


Figure 1.1.: Spare part demand time series.

for the category of parts. The requirements of long-term predictions and of the different spare part classes arise the need of specialized models. Within this work young and fast-moving spare parts are covered. This class of parts is characterized by only a short historic demand period and by regular and frequent demands [34]. IBM already developed a model for this category of spare parts, the Short-Term Prediction Model (STPM). This existing model is at an expansion stage, that allows further refinement, to increase the prediction quality.

Figure 1.1 shows the demand history of a spare part, whose demand will be predicted within this work. The abscissa represents the time and the ordinate shows the spare part demand. The curve on the left side of the vertical line represents the data that is available for training of the model. The area on the right side shall be predicted. It may be noted that this part is a reference part with a much longer demand history than available for the actual parts of the above-mentioned category. Nevertheless, this part once fell into the young and fast-moving spare part class and can now be used for model evaluation. The plot illustrates some of the challenges of this prediction task. Only a few data are available for model training. Based on this information the pattern of the future demand needs to be predicted. The training data not always represents the future pattern. The demand curve underlies plenty of unknown influence

factors. These points could be further extended but they already substantiate that forecasting under these conditions is a tough task, that should be dealt with in this thesis.

1.1. Motivation and Targets

Several works have underlined the economic importance of the spare part business for automotive companies. Klug [57] states that an optimal spare part management is crucial to business success in the automotive industry. Schuh and Stich [82], as well as McKinsey & Company [70], predict a growth of the after sales market, including the spare part business for the next years. Furthermore, Dombrowski and Schulze [29] attest the spare part management a large share of the after sales revenue. All these points underline an economic need of spare part management and its optimization.

Demand predictions are used for many purposes, one example is contract negotiations with suppliers. The more accurate the forecast, the better the starting position for negotiations. Not used spare parts are bounded capital that produces costs by storage and maintenance instead of producing revenues. Therefore, an overestimation must be regarded as negative. Underestimation of demands could lead to bottlenecks in spare part supply. This could in worst case result in unnecessary downtime of the cars, which reduces customer satisfaction and damages the brand overall. According to Klug [57], a good working spare part supply is nowadays an important factor regarding the customers purchase decision. These points summarize the economic need of accurate forecasts of spare part demand for an automotive car manufacturer.

Plenty of works proposed models for spare part demand forecasting. Croston [21] published a model based on Simple Exponential Smoothing for spare part demand forecasting. Syntetos and Boylan [90] enhanced Crostons estimator by adding smoothing parameter. Willemain et al. [99] applied bootstrapping for spare part demand forecasting. Chiou et al. [18] used the grey theory to forecast spare part demands. Hua and Zhang proposed a Support Vector Machine based model and Gutierrez et al. [41] applied a neural network for spare part demand prediction. Most of these works uses statistical models for prediction of the demand. According to Bontempi et al. [8] machine learning approaches obtained promising results in the area of time series forecasting

in the last decade. This trend is not recognizable for spare part demand forecasting, which is a related area.

This motivates the identification of the characteristics of spare part demand time series to perform a literature review for determination of possible computational intelligence approaches, applicable to the spare part demand forecasting problem of this thesis. Artificial Neural Networks stand out by their ability to capture patterns within the data. Proof of Concept tests showed promising results applying neural networks to the spare part demand prediction problem even if there is only few data available for model training. Based on the literature review and the results of the Proof of Concept tests the following research question is phrased for this thesis:

Could an Artificial Neural Network based prediction model forecast the young fast-moving spare part demand with higher accuracy than the currently applied model?

This research question is split up into several parts:

- What computational intelligence models are suitable to forecast the demand of young fast-moving spare parts?
- Can the currently applied model be improved, so that an Artificial Neural Network approach is not needed at all?
- How needs the Artificial Neural Network model be configured to achieve best possible results?

1.2. Structure of Thesis

The next chapter describes the economic fundamentals of spare part management. It introduces the spare part life cycle model that is important for understanding of demand patterns. Further spare part classification possibilities and influence factors for spare part demand are discussed. Chapter 3 introduces the fundamental concepts of time series. Based on an extensive literature review, concepts used for time series and especially spare part demand forecasting are declared and related work is discussed. Furthermore, the fundamental concepts of Artificial Neural Networks are introduced. In Chapter 4 the spare part demand data provided by a large automotive manufacturer is

discussed. Furthermore, the currently applied model is analyzed and possible enhancements are proposed and reviewed. Chapter 5 then proposes the artificial neural network based, especially a deep learning based model for spare part demand forecasting. The potential parameter configurations of the model are discussed and statistically evaluated by plenty experiments to determine the best possible configuration. In Chapter 6 the proposed model is compared to the currently applied model and its suggested enhancements. Finally, Chapter 7 summarizes the findings of the thesis, critically reviews them and provides a research outlook.

2. Fundamentals of Automotive Spare Part Demand Management

This chapter gives a brief overview of the economic fundamentals of spare part management. It defines some of the terms of relevance for this thesis and explains their relationship among each other. The spare part life cycle model as an important background for spare part demand forecasting is introduced. Further different approaches for spare part classification are presented and the scope of this work is restricted accordingly. Finally, some influencing factors of spare part demand are discussed to give a brief introduction into the economically complexity of spare part demand forecasting.

Spare part management is used across all industries. The focus of this work is in the automotive sector and all definitions and explanations could be regarded as of the automotive industry.

Spare Part

Products are generally composed of plenty of parts. As of DIN24420-1 [27] spare parts are "parts, groups of parts (also called components) or complete products, that are needed to replace damaged, worn or missing parts, groups of parts or products." According to Schroeter [81] spare parts are secondary products. They are elements that could be replaced to restore or keep the operating functionality of the primary products during their whole lifetime. This concludes that a spare part demand could only exist after the purchase of the primary products, which are cars in case of this thesis. Strunz [88] declares spare parts as elements that get worn during the usage of the primary product and need to be replaced, and states this action as fundamental activity of the maintenance process.

According to DIN31051 [28] spare parts could be further differentiated into backup parts, usage parts and small parts. Strunz [88] describes backup parts as parts that are kept for a potential part failure of a particular primary product, usage parts as parts that are typically worn during the usage, depending on the intensity of the usage and small parts as universal, often standardized parts of small value.

Based on the origin spare parts could be classified into the following three groups [57]:

- **Original spare parts** are parts that are produced from the Original Equipment Manufacturer (OEM).
- **Foreign spare parts** are identical parts produced from other manufactures than the OEM.
- **Used spare parts** are used, recycled or refurbished parts.

In the context of this work all spare parts are OEM parts. Further possibilities of spare part classification are covered in Section 2.2 about classification of spare parts.

Spare Part Management

According to Biedermann [7] spare part management or spare part logistics deals with all management activities, which assure that a spare part is at the right time, in the right quality and quantity at the right place at minimal costs. Klug [57] adds that the spare part management connects all activities around maintenance and spare parts. As of Schuh and Stich [82] it is the target of the spare part management to control all involved processes in the right way to accomplish an economically optimized spare part stock. Schroeter [81] supplements that due to the high complexity and uncertainty of spare part demand estimation often a security stock is kept buffering potential underestimations. This influences the capital commitment costs. An optimal spare part stock tries to minimize the security stock, and this results in less fixed capital, minimized costs for storage and if estimated correctly, still in a minimization of downtimes. Nevertheless, the determination of an optimal spare part stock is a nontrivial process and includes plenty influence factors. Furthermore, Schuh and Stich [82] state that the spare part management could be regarded from two different perspectives. On the one hand side from the viewing point of a

customer and on the other hand side it is regarded from the viewing point of a manufacturer. The latter one is the perspective used for this thesis. Klug [57] adds that an effective spare part management has also influence on the customer satisfaction because in an ideal case there is nearly no downtime of the product.

After Sales Services

The spare part management is part of a car manufacturer's after sales services. According to Klug [57] the after sales services are a marketing tool that includes all activities to increase the customer retention after a purchase. Customers should be satisfied, and the customer loyalty of the brand should be strengthened. Vahrenkamp and Kotzab [94] add the fact that a high degree of service can be a criterion for a product decision at all or regarding future decisions. Therefore, the period of after sales, especially the spare part management, involves a high potential of customer retention. Satisfied and convinced customers potentially recommend the brand, which also has a positive influence regarding new customers. As of Pfohl [75] there is also feedback from service entities that could be used for improvement of the after sales services or even for future designs. Klug [57] therefore concludes that the after sales services are nowadays an important competitive differentiation for car manufacturers.

According to Schuh and Stich [82] the profit of the after sales is steadily increasing over the last years. They have become an important area for car manufacturers. The after sales services market contains high potential profit margins and is often more profitable than the primary product market. Figure 2.1 shows the proportion of the worldwide profit of car manufacturers from 2014 in billions of Euro and underlines the importance of after sales services. The earnings of the after sales exceeds the return of new car sales by a factor of over 9. Between 50% and 70% of the total after sales revenue of a car manufacturer are generated by the spare part business state Dombrowski and Schulze [29]. According to a study of McKinsey & Company [70] the global market value of automotive aftermarket will grow from approximately 760 billion USD in 2015 to 1200 billion USD by 2030. Therefore, the spare part sector will even grow in business importance.

Inderfurth and Kleber [50] noted in their paper that the lifetime of a car lasts usually at least fifteen years, often longer. As stated by Hagen [43] according

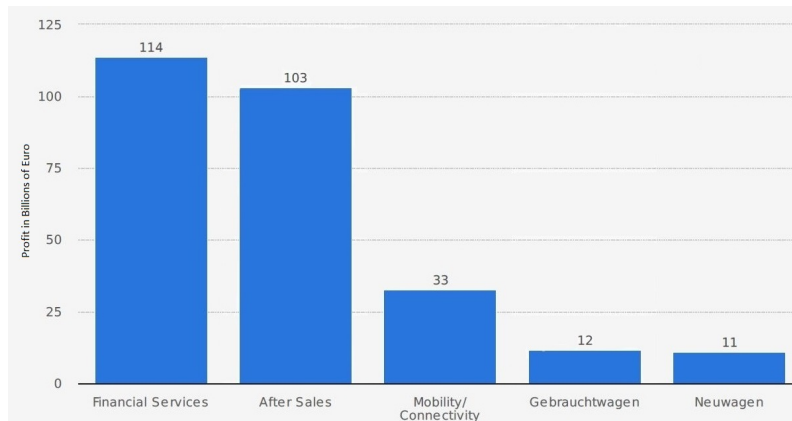


Figure 2.1.: Worldwide profit of car manufacturers 2014 [86].

to legal requirements car manufacturers are forced to provide spare parts for their products for a period of ten years after the end of production. Klug [57] found most OEMs to use this requirement to their marketing benefit and extend this period of after sales services to an average time span of 15 years after the end of production. He also added that this long period of spare part supply results in high bound capital and storage costs, which underlines the need of optimization of spare part demand estimation.

2.1. Spare Part Life Cycle Model

To understand and predict the spare part demand the life cycle model of spare parts is of high importance. Figure 2.2 shows the life cycle model of a spare part, also called the all-time pattern. Based on the work of Fortuin [33], Klug [57] describes the model in detail, where different phases for the spare part demand could be derived. Dombrowski and Schulze [29] state that the model assumes, that the primary product and the spare part demand follow some rules from the beginning of production until the end of life of the primary product. The demand pattern of a spare part is always related to the demand of its primary products, which is related to the cumulated sales of this. As of Hagen [43] the model assumes also an ideal-typical demand pattern, which is not always the case in reality. Nonetheless, this does not reduce the significance of the life cycle model.

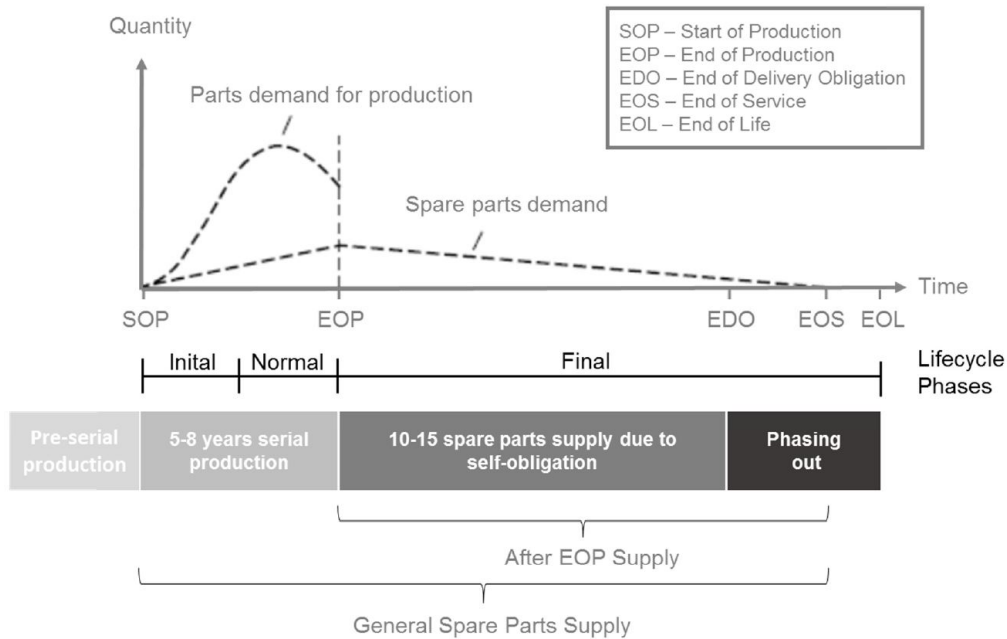


Figure 2.2.: Spare part demand life cycle model [57].

There are some relative dates in the life cycle of a spare part that are used to describe the phases of its life [32]:

- **Start of Production (SOP):** At the SOP the production of the primary product begins.
- **End of Production (EOP):** The serial production of the primary product ends.
- **End of Delivery Obligation (EDO):** The warranty related, supplier contract related or self-obligated spare part availability ends.
- **End of Service (EOS):** The service of the primary product by the OEM ends. OEM spare parts are no longer distributed.
- **End of Life (EOL):** The primary product and the spare parts disappear from the market.

Based on the above defined dates basically three major life cycle phases with different impact on the spare part demand are distinguished by Klug [57]. The absolute dates of these phases differ in literature from author to author. Despite the impact of the exact dates is relatively small for this thesis, the concept of Klug [57] is seen as the most important.

Initial Phase

The beginning of the initial phase is the SOP, a new car reaches the market. This phase ends within the first third until half of the serial production period. Klug [57] states the difficulty to estimate the spare part demand besides the used parts for the primary product due to the lack of historic knowledge about spare part failure rates and demand patterns as characteristic for this phase. Nevertheless, to ensure an unimpaired service level high security demands are stocked, as stated already in the work of Fortuin [33]. Klug [57] describes that these security stocks are used for immediate reaction to keep the image of the product on a high level. Often these security stocks are overestimated and involve an optimization potential. Schroeter [81] notes, that it is also beneficial to be able to forecast the demands in terms of manufacturer contracts and capacity planning.

All parts that are used to forecast demands within the scope of this thesis are in the initial phase of their lifetime.

Normal Phase

The second phase lasts from the end of the initial phase until the EOP of the primary product. According to Klug [57] this phase is characterized by a stabilized demand of the primary product. The OEM has already gained some knowledge about the parts used in the car. Klug [57] also notes that the market consistency of the primary product doesn't result in the demand patterns of the spare parts. Due the today's high complexity of cars, which results for an OEM in a broad spectrum of parts, the ever shorter innovation cycles, long spare part warranty periods and the random nature of part failure it is still difficult to estimate the spare part demands precisely during this phase but the forecasts are already more accurate than in the initial phase, as stated by Klug [57] and also by Schroeter [81].

Final Phase

The final phase begins with the car's EOP and lasts until the EOL. According to Klug [57] the main characteristic of the final phase is the steadily decreasing primary product stock in the market. Fortuin [33] notes that the production of parts is reduced to the aftermarket demand and is abandoned for plenty of

parts over time. Due to the decreasing part demand the production of expiring parts gets more and more expensive according to Schroeter [81]. Foreign parts get an increasing market share. The production of parts that are not used in other car models becomes unprofitable, which results in the end of their manufacturing. Klug [57] underlines that during this phase a strategy to satisfy all delivery obligations, e.g. because of warranty periods, needs to be chosen. To handle spare part demands in the time after the end of part production the OEMs often use the all-time requirement, where the spare part demand until the end of life is estimated and spare parts are stocked accordingly as stated by Fortuin [33] and by Klug [57].

2.2. Classification of Spare Parts

Spare parts are differentiated in literature in many ways. First possibilities, e.g. based on the origin of the spare part, were already introduced in the spare part definition and the spare part lifecycle. According to Loukmidis and Luczak [67] not every prediction technique is applicable to every type of demand pattern. Because each class of spare parts has its own characteristics, a specialized demand forecasting approach should be applied to each. This specialization of the prediction technique results in the need of spare part classification as stated by Klug [57].

Based on the categorization criteria some of the most common used and for this work important classifications are discussed in the following. An overview of classification approaches for spare parts is given in Figure 2.3.

One of the most common classification techniques, according to Bacchetti and Sacconi [3], is the ABC analysis. Schuh and Stich [82] describe the classification as based on the relevance of the spare parts for the company. This method tries to estimate the revenue value share of the parts and their demand patterns to classify them either as A, which make about 80% proportion of the overall spare part revenue value, class B with about 15% share of the revenue value and C with the remaining 5%. Klug [57] adds that the ABC analysis makes use of the Pareto principle and the Lorenz curve. An enhancement exists in the XYZ analysis, which adds a demand regularity based approach as described by Loukmidis and Luczak [67]. It uses features of the demand predictability for the classification scheme. Parts of class X are easy to predict, parts of class Y are characterized by an unstable demand, which makes them more difficult

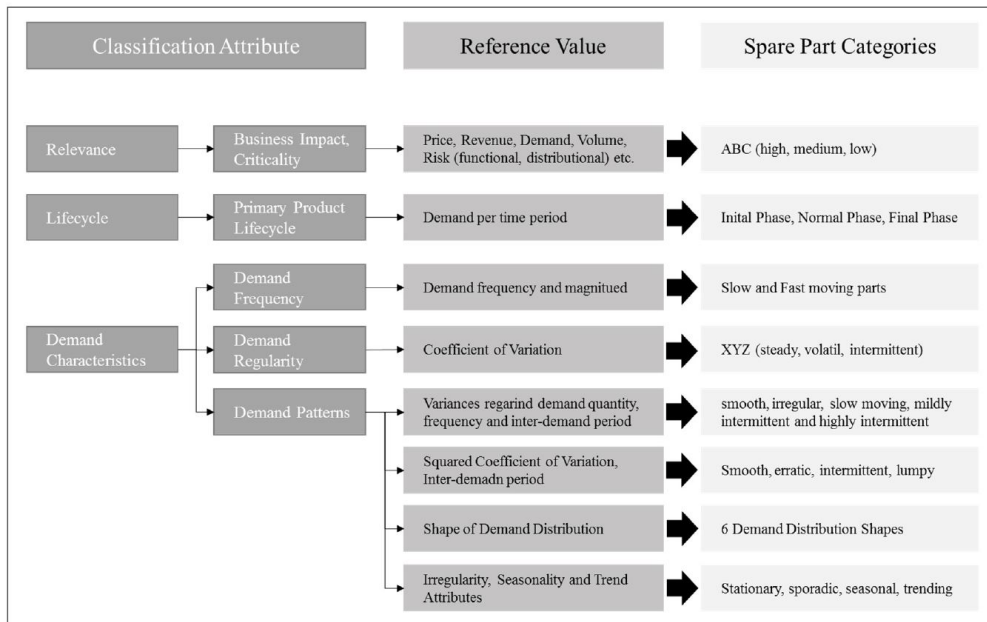


Figure 2.3.: Spare part classification approaches [66].

to predict and class Z parts are very difficult to forecast already within a short horizon because of their chaotic demand pattern. The combined analysis results in nine different classes. Additionally Schuh and Stich [82] noted that there exist also modifications, which make use of the demand frequency instead of economic relevance in terms of revenue value as ABC classification features.

It is also possible to categorize spare parts based on the demand characteristics. One popular approach was published by Boylan et al. [11]. Based on the mean inter-demand interval, that averages the interval between two successive demand occurrences, the mean demand size and the coefficient of variation of the demand sizes, this approach sets up six different classes: intermittent, slow moving, erratic, lumpy and clumped. Fortuin and Martin [34] explain a general distinction based on the demand frequency over a period in two main classes, which are slow-moving and fast-moving parts, which also is a widely used approach. Slow-moving spare parts are characterized by an irregular and infrequent demand. Fast-moving parts on the opposite have a regular and frequent demand.

In the scope of this work this approach is used. Furthermore, only fast-moving spare parts are covered through the model.

Further categorization influence factors that exceed the scope of this work are the costs in case of a failure of the primary product, the spare part logistic costs, the cost for storage of parts, the costs for acquisition of parts and the replaceability of the parts. The interested reader is referred to the work of Bacchetti and Saccani [3], as to the book of Loukmidis and Luczak [67] and to the work of Schuh and Stich [82] for a detailed review of spare part classification approaches.

2.3. Influence Factors for Spare Part Demand

As pointed out by Loukmidis and Luczak [67] spare part demand is influenced by many different factors, each with different impact. Literature generally distinguishes between influence factors related to the primary product, related to the spare part itself, factors related to maintenance and influence factors related to the spare part market as well as other exogenous factors.

According to Loukmidis and Luczak [67] spare part demand is by its nature a derivative need. The demand is strongly related to the number of primary products purchased. The more primary products are on the market, the higher the spare part demand potential. Furthermore, Pfohl [75] states, that the age structure and the utilization intensity of primary products in use influence the demand. Also, lifetime, exploitation and recycling of the cars after the end of usage affect the spare part demand pattern. If it comes to demand forecasting planned sales of the primary product are also influencing factors of the primary product to be considered as noted by Klug [57].

The second class of influence factors is part related. According to Loukmidis and Luczak [67] the major factor is the estimated lifetime of the part. It is generally dependent on the type of the part, the utilization intensity and on the type of use. Furthermore, Klug [57] adds that the composition of the primary product of standard parts, modules or specialized parts has an influence on the demand pattern. In the scope of forecasting the known failure rate of the parts, the security stocks and the demand history additionally influence the future demand as stated by Pfohl [75].

Furthermore, Loukmidis and Luczak [67] mention that the strategy of maintenance also influences the spare part demand. Either maintenance could be done on a regular basis, so to say preventive, it also could be done based

on the usage or condition of the primary product, or maintenance could be done only in case of failure. Each strategy has different influence on the spare part demand. Klug [57] notes that usually a mixture of these strategies is applied in reality, which results in a mixture of stochastic and deterministic demand influence factors. According to future demands historic knowledge of the maintenance influence, e.g. service intervals, can affect the demand as well.

Finally, Loukmidis and Luczak [67] point out that the spare part portfolio on the market has influence on the spare part demand. Parts offered from different vendors than the OEM or from different sources, like recycled or refurbished parts affect the demand pattern. The purchase of a new primary product instead of maintenance also has its share, related itself by the age structure of the primary products. Pfohl [75] adds that new technologies and upgrades or changing legal requirements have an influence too.

These are only the most important influence factors and by far not all of them. Interested readers are referred to the work of Loukmidis and Luczak [67] as a starting point. Regarding the above discussed factors, Klug [57] notes that the estimation of spare part demands already becomes a very complex task. Plenty of the factors are hidden and cannot be made visible exactly. Also, the influence of each of these factors is not clearly derivable for each demand. In the scope of this work the historic demand pattern, the historic primary product sales and the planned car sales are used for demand forecasting of spare parts within the initial phase.

3. Fundamentals of Time Series and Spare Part Demand Forecasting

This chapter provides an introduction to the area of time series forecasting and in particular, spare part demand forecasting. First some basic terms and principles are defined, and special characteristics of time series are reviewed. Then the concepts of spare part demand forecasting, from the early beginnings until today's machine learning approaches are introduced. One of the most recently emerging approaches, the Artificial Neural Network (ANN) model for time series forecasting is reviewed in more detail. Furthermore, the for this work relevant concepts of Recurrent Neural Networks and deep learning for time series forecasting are discussed.

3.1. Definitions

This section defines the basic concepts, common to all approaches of time series and spare part demand forecasting. It builds the basis for the later work.

Time Series

Palit and Popovic [73] define a time series as a series of values, observations or measurements x_1, x_2, \dots, x_t that is sampled or ordered sequentially by a feature of time. The data is indexed by time with equal distance Δt . Chattfield [16] adds, that the measurements can be taken continuously through time in case of a continuous time series or at discrete time steps in case of a discrete time series. The values itself can be either continuous or discrete. Often continuous time series are converted to discrete time series by sampling in discrete time

intervals. The frequency is called the sampling rate. Typically, the data of discrete time series is distributed over equal time intervals. It is also possible to aggregate the data over a period of time, e.g. daily data can be aggregated by weeks or months. Laengkvist et al. [64] noted, that the values of the time series usually are composed of a deterministic signal component and a stochastic noise component, originating from the measurement, corrupting the series. Often it is not clear if the available information is enough to fully understand the generating process and its included dependencies. In the scope of this work only discrete time series are of relevance and all further descriptions are related to discrete time series.

Besides the time as main feature, time series are characterized by linearity, trend, seasonality and stationarity, as described by Palit and Popovic [73].

- **Linearity** indicates, that the time series could be represented by a linear model, based on the past and present data. Time series that cannot be represented by a linear function are called non-linear. In real scenarios both types are often mixed, e.g. a time series shows local linearity but global non-linearity, which makes according to Palit and Popovic [73] a differentiation and appropriate model selection difficult.
- **Trend** is described by Chatfield [16] as a long-term decrease or increase in the mean level of the time series. Long-term covers a period of several successive time steps and is not clearly defined in literature, as well there exists no fully satisfying mathematical definition for trend. Palit and Popovic [73] add, that the decomposition of variation into seasonal components and trend components is handled differently in literature, mostly originating from the difficulty to separate the pure time series signal from the influences of seasonality, trend and noise.
- **Seasonality**, as defined by Chatfield [16], characterizes the periodical fluctuating behavior of a time series. Similar patterns repeat at certain periods of time with varying influence. Additive seasonality is independent of the local mean level, the mean of a short period of time, and multiplicative seasonal variation is proportional to the local mean level. This means for example in case of an upward trend, the variation influence because of seasonality also increases.
- **Stationarity** describes the behavior of the mean and the variance of the time series data, as defined by Chatfield [16]. If both values are nearly

constant over time the series is called stationary, else it is called non-stationary. Palit and Popovic [73] mentioned, that stationary time series are characterized by a flat looking pattern with small influence of trend or seasonality.

As of Chatfield [16], time series can be further distinguished according to the number of predictor features. Univariate time series sample only a single time dependent process. Multivariate time series are composed of more than one feature. Each point in time is described by simultaneously sampled values from each of the underlying time dependent processes.

Time Series Forecasting

According to the book of Chatfield [16], time series forecasting tries to compute future values of a time series based on the observed present and past data. It is part of the area of predictive analytics. Given a time series x_1, x_2, \dots, x_t forecasting means to compute future values, such as x_{t+h} . The positive integer h is called the lead time or forecasting horizon. The forecast at time t for h steps ahead is denoted by $\hat{x}_t(h)$. The case of $h = 1$ is called one step ahead forecast and all cases for $h > 1$ are called multi-step or h -step ahead forecast. If h defines a range it is called a range forecast. Forecasting methods can be distinguished in objective forecasts, univariate forecasts and multivariate forecasts.

- **Objective forecasts**, as described by Chatfield [16], are based on the judgement of experts and their knowledge. These forecasts include a subjective bias. A popular approach is the Delphi-method [22], where experts are surveyed in several rounds and the estimations are combined to a forecast.
- **Univariate forecasts** are based on a time series originating from a single underlying process, as defined by Chatfield [16]. Palit and Popovic [73] mention, that a model based on a univariate time series tries to extrapolate the pattern from the generating process.
- **Multivariate forecasts**, according to Chatfield [16], take into account more than one time defined process for forecasting. Palit and Popovic [73] further describe, that each generating process has its own influence on the time series. A multivariate model tries to combine the generating

processes, to estimate the time series pattern and to derive the influence of each underlying process.

Chatfield [16] argues, that except of some special cases, usually statistical approaches are superior to objective forecasts. Often the above mentioned classes are combined to use the best from each world for forecasting, e.g. expert knowledge is included into a multivariate forecasting model. Palit and Popovic [73] mention, that time series forecasting can be further classified based on the complexity of the approach or on the human interaction need. In the scope of this work only univariate and multivariate forecasting approaches are used and further descriptions are only related to these two classes.

3.2. General Spare Part Demand and Time Series Prediction Models

Time series forecasting can be applied in different areas. If the to be forecasted series is composed of timely dependent spare part demands it is called spare part demand forecasting. As of Callegaro [13], first methods used were classical statistical models originating from economics and time series modeling, with no specialization for spare part characteristics, like Simple Exponential Smoothing (SES), Autoregressive (AR) models or Moving Average (MA) approaches, as combinations and modifications of these, e.g. like models of the Autoregressive–Moving-Average (ARMA) family. Bartezzaghi et al. [4] noted, that all these methods assume a certain degree of stability in the environment, which is often not given for spare part demand time series. According to Boylan and Syntetos [10], this ignorance of particular properties of demand series led to substantial overestimation of future demands and to too small forecast horizons that could be predicted with a sufficient degree of accuracy. Because of the need of accurate forecasts in plenty of areas researchers began to develop approaches specialized for spare part demands.

In the following some of these specialized models, but also general time series prediction approaches applicable for spare part demand are discussed. The above mentioned classical statistical models would exceed the scope of this work. The interested reader may be referenced to the work of Callegaro [13] for an extensive overview of statistical models used for demand forecasting.

3.2.1. Statistical Models

One of the first, and for a long time most widely used, approach developed was Crostons Method [21]. He proved that SES overestimates lumpy demand because the latest time step gets the highest weight. This results in a high forecast after demand occurred, even if in the next time step no demand occurs. To solve this problem, he constructed a SES based model, composed of the size of the demands and the average interval between demand occurrences. A forecast based on Crostons Method is calculated with the following recursive formula:

$$\hat{z}_{t+1} = \hat{z}_t + \alpha(x_t - \hat{z}_t) \quad (3.1)$$

$$\hat{p}_{t+1} = \hat{p}_t + \alpha(q_t - \hat{p}_t) \quad (3.2)$$

$$\hat{x}_{t+1} = \frac{\hat{z}_{t+1}}{\hat{p}_{t+1}} \quad (3.3)$$

α is the smoothing parameter, x_t is the demand at time t and q_t is the time distance between the occurrence of the current and the previous demand. \hat{z}_t represents the exponential smoothed demand. \hat{p}_t equals the positive demand interval at time t , forecasting the time step with the next demand occurrence by SES. Both are only updated in case of a demand occurrence. A difficulty of Crostons Method is to choose an appropriate α value.

Syntetos and Boylan [89] showed 2001 that Crostons Method is biased, depending on the smoothing parameter. They provided an extension of the original method, which is known as Syntetos-Boylan Approximation (SBA) [90]. To deal with the bias an adapted smoothing parameter is added to Crostons Method and the forecast is calculated as of Equation 3.4. With extended simulation experiments on 3000 stock keeping units from the automotive industry, Syntetos and Boylan showed the superiority of their approach, compared to Crostons Method, MA and SES. The difficulty of choosing an appropriate smoothing parameter value remains in their enhanced approach.

$$\hat{x}_{t+1} = \left(1 - \frac{\alpha}{2}\right) \frac{\hat{z}_{t+1}}{\hat{p}_{t+1}} \quad (3.4)$$

Another statistical method used for time series forecasting is bootstrapping. The basic bootstrapping approach was published by Efron in 1979 [30]. It is a sampling technique to calculate statistical measurements from an unknown

underlying distribution by taking plenty samples with replacement and aggregating the statistics over each sample. Bootstrapping was applied to forecasting of intermittent spare part demand by Willemain et al. [99]. They modified the approach to take spare part characteristics into account and evaluated the proposed model on nine industrial data sets against SES and Crostons Method to show the approaches superiority. Gardner and Koehler [36] criticized the results according to the experimental methodology which is questioning the model at all. Later Porras and Dekker [76] applied the approach, proposed by Willemain et al. to spare part demand data of a large oil refinery with promising results. Nonetheless, the research interest according to bootstrapping and spare part demand forecasting is decreasing.

Furthermore, a statistical model used for spare part demand forecasting is the Grey Prediction Model (GPM). It is motivated on the Grey theory developed by Deng [25]. The GPM is based on the Grey generating function $GM(1, 1)$, a time series function that uses the variation in the underlying system to find relations between the sequential data. Interested readers are referred to the work of Deng [26] for details on the theory of the GPM. The approach is characterized by the ability of forecasting with limited amount of data and requires no prior knowledge of the time series. Chiou et al. [18] used the GPM to forecast spare part demands. They state the Grey forecasting model to be superior for short term predictions, compared to other (unnamed) time series models and SES, but for the mid and long term not suitable.

In 2011 Lee and Tong [60] published a modified version of the GPM. They augmented the model by incorporating genetic programming. By experimental evaluation on the energy consumption of China Lee and Tong showed the superiority of their approach, compared to the basic GPM and simple linear regression. Hamzacebi and Es [44] applied a parameter optimized GPM for forecasting the annual energy consumption of Turkey. The optimized GPM was evaluated against the basic model. The proposed approach outperformed the classical GPM and also increased the forecast accuracy for the midterm.

3.2.2. Machine Learning Approaches

Besides the methods based on statistical models Bontempi et al. [8] noted that machine learning approaches gained more research attendance in the last decades. In the following some of these models are discussed.

Support Vector Machines

Support Vector Machine (SVM) models are one of these computational intelligence techniques frequently used for time series forecasting. This approach is based on a paper by Vapnik et al. [95]. A SVM creates a hyperplane to linearly separate the data into classes by placing the hyperplane between the data-points. The distance of the data-points to the hyperplane is maximized by constrained based optimization. To deal with non-linear separable problems a so-called kernel trick is applied. The data is transferred to a higher dimensional space, where the problem becomes linear separable. By adjusting the constraint based optimization according to a generalization of the data-points instead of the maximization of the margin between the classes, SVMs could also be used for regression problems, like time series forecasting. If used for regression problems they are sometimes called Support Vector Regression (SVR).

In 2003 Cao and Tay [14] used a SVM model to forecast financial time series. They compared the forecast performance of a SVM model against a Multi Layer Perceptron (MLP) neural network and against a Radial Basis Function (RBF) neural network on five real world financial data sets. In all but one case the SVM outperformed the other models. They explained the superiority of the SVM by the fact that this model finds the global optimum of the optimization, whereas the ANN could get stuck in local optima. Furthermore, an extended version of the SVM model with adaptive hyperparameters, parameters that are set before the learning of the actual parameters by the machine learning approach takes place, for handling the non-linearity of financial time series is proposed. This enhanced model outperformed the classical SVM approach on the evaluated data sets, but adds complexity by setting the hyperparameters correctly.

Hua and Zhang [48] proposed a hybrid SVM approach for intermittent spare part demand forecasting in 2006. They used the SVM model to forecast the occurrences of nonzero demands and integrated this information with explanatory variables into a composed model. Experimental results on 30 real world data sets from the petrochemical industry showed that their proposed approach outperformed SES, Crostons Method and the basic SVR. They also stated that their approach is suitable for scenarios with limited historical information.

Another approach using SVM to predict short-term traffic flow was published by Lippi et al. [63] in 2013. To deal with the high seasonality of the traffic flow

time series a seasonal kernel, to capture repeating patterns, was used in their model. Experimental evaluation was performed on data from the California Freeway Performance Measurement System. The seasonal kernel SVR was compared against several other approaches, like AR models, ANN and SVM models with different kernels. Based on the experiments the seasonal AR with integrated MA (SARIMA) performed best, but the seasonal kernel SVM was found competitive to the computational expensive superior models. They also confirmed that the seasonal pattern is a key feature for time series forecasting.

In the same year Kazem et al. [54] published a paper about SVM to forecast stock market prices. They proposed a 3-fold model. To overcome the non-linearity of these time series a phase space reconstruction, originating from dynamic systems theory, is applied as a data pre-processing step. In the second step the hyperparameters of the SVM are optimized by a chaotic firefly algorithm, a nature inspired meta-heuristic optimization algorithm. In the last step the SVM is trained to forecast the stock market prices. Due to the iterative behavior of the approach it is computational expensive. An experimental evaluation against ANN and basic SVM showed the superiority of the proposed model.

An ensemble model of SVM for building energy consumption forecasting was proposed by Zhang et al. [101]. The hyperparameters of each SVM and the weights for each ensemble member are optimized with Differential Evolution (DE), an evolutionary optimization algorithm. Experimental evaluation was performed with different optimization algorithms for hyperparameter estimation and each member of the ensemble was compared against the proposed model, which outperformed all separate components. Unfortunately, the proposed approach is not evaluated against other models.

In 2017 Kanchymalay et al. [52] published a paper about multivariate time series forecasting by SVM. They choose nine different features to represent the time series and evaluated the forecasting performance of SVR against MLP and Holt-Winter exponential smoothing. The experimental evaluation on a crude palm oil price data set showed that SVM slightly outperformed the MLP and was clearly superior to Holt-Winter exponential smoothing.

The above-mentioned works are by far only an excerpt of the extensively used SVM model. Interested readers are referred to the works of Cheng et al. [17] and Deb et al. [23], both including an extensive literature review as a starting point.

Fuzzy Models

Another class of computational intelligence models, used for time series forecasting, are Fuzzy time series. The concept was introduced by Song and Chissom [85] in 1993. The time series of this model are represented by fuzzy sets in a universe of discourse, corresponding membership functions and fuzzy logical relations of different order. Singh [83] used Fuzzy time series in 2007 to forecast wheat production. He evaluated his model on two real world data sets and showed its competitiveness. Pei [74] used fuzzy time series for energy consumption predictions. He improved the classical model by extending the fuzzification by a K-Means algorithm. The proposed approach showed a higher forecast accuracy on the evaluated data set. Nonetheless, all fuzzy models require a high degree of expert knowledge for defining the universe of discourse and for definition of the fuzzy rules describing the relations.

Hybrid Models

Hybrid models composed of ideas from the above mentioned approaches and other machine learning algorithms are used for time series forecasting as well. According to Deb et al. [23] these models try to combine advantages of the involved algorithms and are usually more robust. These enhancements are often bought by computational expensiveness and algorithmic complexity. In the area of spare part demand forecasting the already described hybrid SVM model suggested by Hua and Zhang [48] shall be mentioned here too. Furthermore, Lin et al. [62] proposed a hybrid model, composed of elements from ANN, fuzzy systems, evolutionary and cultural algorithms. They evaluated their model on three chaotic time series, a special kind of non-linear time series, against other evolutionary models and showed its superiority. Nonetheless, a comparison against typical time series forecasting approaches is missing, so no conclusion about the revenue of the highly complex approach can be drawn. Ravi et al. [77] suggest a model composed of elements from chaotic systems, MLP and multi-objective evolutionary algorithms, to predict financial time series. The proposed model was evaluated on four financial real world data sets and showed promising results according to forecast accuracy.

The above discussed hybrid models exemplary should show the manifold possible combinations of approaches. A more extensive review of hybrid approaches,

by far would exceed the scope of this thesis. The work of Deb et al. [23] is recommended for an overview.

The discussed models are frequently used approaches for time series forecasting. All of them have their own strengths, weaknesses and specialties. The basic statistical models, Crostons Method and Syntetos-Boylan Approximation are easy to compute and both later ones are designed especially for spare part demand. Nonetheless, research showed that plenty machine learning approaches outperform these models according to forecast accuracy. Other statistical models add complexity and often showed promising results only for particular time series problems. Support Vector Machines as widely used machine learning approach protrude by the optimal solution found, what makes them competitive against all other models. Nevertheless, hyperparameter derivation is a nontrivial process and computation can get complex. Fuzzy time series feature a great descriptive power but require a lot of expert knowledge. Hybrid models are usually effective for a particular problem but often add high computational complexity. In the next section another widely used machine learning approach, the Artificial Neural Network is discussed in detail.

3.3. Artificial Neural Networks for Time Series Forecasting

In the following the fundamental concepts of Artificial Neural Networks are introduced. The basic principle of an ANN is explained and its components are discussed. A literature review underlines the importance of ANNs for time series forecasting. Furthermore, the concepts of Recurrent Neural Networks are introduced and relevant literature is reviewed. Finally, deep Artificial Neural Networks are discussed.

3.3.1. Fundamentals of Artificial Neural Networks

According to Mitchell [71] Artificial Neural Networks are partly biological inspired mathematical, massively parallel, supervised learning models, containing layer-wise organized units, so called neurons, that are connected. Each connection directs from the output of a neuron to the input of a neuron and

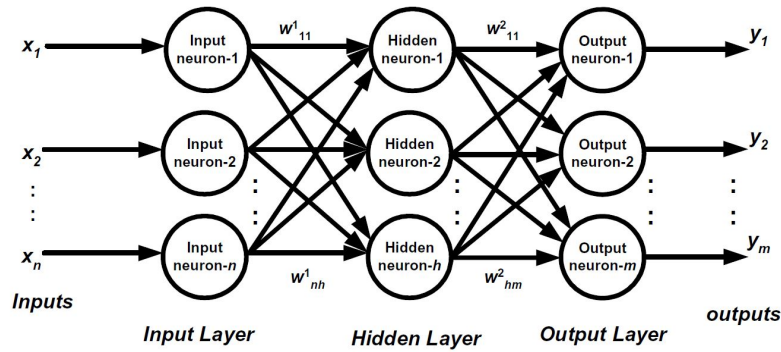


Figure 3.1.: Artificial Neural Network Model [73].

has a variable weight assigned. The model could be represented by a directed, weighted graph. The input is processed from the input-layer to the output-layer via several optional hidden layers. Each neuron calculates its output by an activation function and passes the result to the next neuron, until the output-layer is reached. The parameters of the model, e.g. the particular weights of the connections, are learned during a training phase. An exemplary graphical representation of the model, in particular of a Multi Layer Perceptron, a special kind network, also called feed forward network, where each neuron of a layer is connected to each neuron of the next layer, can be found in Figure 3.1. The number of successive layers is called the depth of the network, the number of units per layer are called the width of the network. The overall structure including the depth, width, types of layers or units and how they are connected is defined by the topology, or architecture of the ANN. According to Palit and Popovic [73], ANNs have been successfully applied to problems of signal analysis, classification, pattern recognition, feature extraction and many more. Among other things, they are characterized by the ability of capturing functional relationships among the data, universal function approximation capabilities and the ability of recognizing non-linear patterns in the data.

Figure 3.2 shows the model of a single neuron, in particular a Perceptron. The Perceptron, originating from a paper by Widrow and Hoff [98], is one of the most widely used basic units of an ANN. The outcome of the Perceptron is calculated according to Equation 3.5. The weighted inputs and a bias, representing a threshold value, are used to calculate the output of the summing element, $v = \mathbf{w}^T \mathbf{x} + w_0$. It may be noted that bold lower case characters are representing vectors. The result of the nonlinear element is generated by

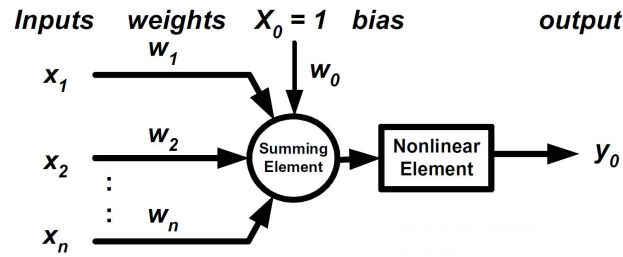


Figure 3.2.: Model of a Neuron: Perceptron [73]

the unit step function defined in Equation 3.6, which is applied as activation function. This means, that the Perceptron is only activated, sometimes also referred to as firing, in case of $v = \mathbf{w}^T \mathbf{x} + w_0 \geq 0$, which is controlled by the learned weights and the bias.

$$y_0 = f\left(\sum_{i=1}^n w_i x_i + w_0\right) \quad (3.5)$$

$$f(v) = \begin{cases} 0 & \text{for } v < 0 \\ 1 & \text{for } v \geq 0 \end{cases} \quad (3.6)$$

Activation Functions

According to Palit and Popovic [73], the sigmoid activation function, as shown in Equation 3.7 was widely used for a long time since the early days of ANN. As of Goodfellow et al. [40], the preferred activation functions changed over time and specialized functions were developed. In the scope of this work further the Rectified Linear Unit (ReLU), as defined in equation 3.8, is used. Glorot et al. [39] found, that the ReLU activation function is superior for the training of more complex networks than the Sigmoid function. Furthermore, a modification of this function, the leaky ReLU, as defined by equation 3.9 is used in the scope of this work. According to Maas et al. [69], leaky ReLU adds a small gradient, even if the unit is not active. Last but not least the SoftPlus activation function will be used, as defined by Equation 3.10. Glorot

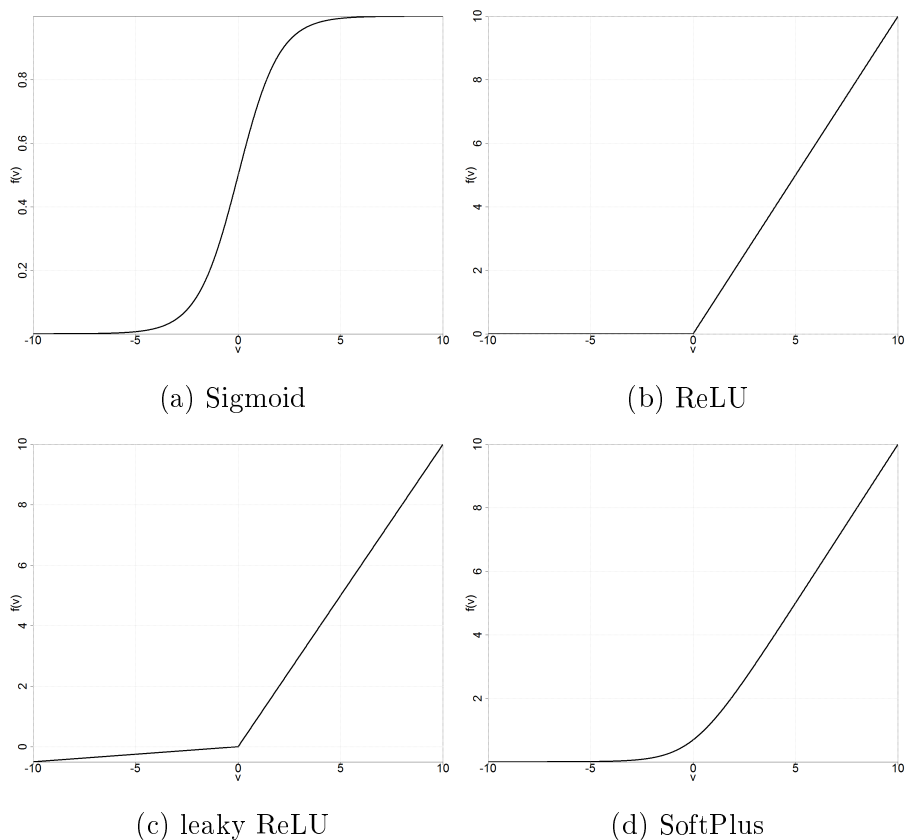


Figure 3.3.: Activation Functions $f(v)$.

et al. [39] describe this function as a smoothed version of the ReLU activation function, which results in a different behavior of the gradient based learning.

$$f(v) = \frac{1}{1 + \exp(-v)} \quad (3.7)$$

$$f(v) = \begin{cases} 0 & \text{for } v < 0 \\ v & \text{for } v \geq 0 \end{cases} \quad (3.8)$$

$$f(v) = \begin{cases} 0.01v & \text{for } v < 0 \\ v & \text{for } v \geq 0 \end{cases} \quad (3.9)$$

$$f(v) = \ln(1 + \exp^v) \quad (3.10)$$

An graphical overview of the four activation functions is given in Figure 3.3. It may be noted that the leaky ReLU factor of 0.01 in case of $v < 0$ was changed to 0.05 for plotting, as of visualization purposes. Nonetheless, these are only

a few of common activation functions. Interested readers are referred to the book of Goodfellow et al. [40] for an overview.

Learning for Artificial Neural Networks

During the training phase the learning of the weights as a supervised learning process is performed. According to Palit and Popovic [73], the Backpropagation algorithm is the most widely used learning approach. As of Schmidhuber [80] the concepts originate back to the 1960s and 1970s. The approach gained popularity after the publication of the paper of Rumelhart et al. [79] in 1986. As of Palit and Popovic [73], the principle of the algorithm can be described in the following way: While the training data is processed through the network in a forward direction, the error of the network is computed based on the output value of the ANN and the output intended by the data. This error is then propagated in backward direction, from the output- to the input-layer of the network, to adjust the weights of the connections accordingly. The Backpropagation algorithm is used to calculate the gradient, which in turn is used for optimization of the weights. This approach is applied in an iterative way, several times for the whole training data set. The number of iterations is called the training epochs of the ANN. The training could be finished after a fixed number of epochs or if the error has reached a lower bound, e.g. by an approach called early stopping.

Optimization Approaches

Different optimization algorithms are used for calculation of the weights. An introductory overview can be found in the work of Schmidhuber [80] or in the paper of Ruder [78]. In the scope of this work three different approaches are of importance: Stochastic Gradient Descent (SGD), RMSprop and Adam. According to Bottou [9], SGD is nowadays one of the most used optimization algorithms in the area of ANN, therefore it can be understood as the general-purpose optimization approach in the area of neural networks. Instead of precisely computing the gradient based on all training samples at once it estimates the gradient for each epoch in an iterative way based on the currently picked sample z_t . The gradient is calculated by Equation 3.11, where $Q(z, w)$ is an error function, e.g. the mean squared error, given the current sample and a particular parameter set w . According to SGD, the weight is updated as

stated in Equation 3.12, after each sample was processed. t indicates the training epoch and η is the learning rate, controlling the speed of convergence. It may be noted, that the hyperparameter η needs to be adjusted carefully. Too large η values lead to oscillation, so the (local) optimum is not reached and too small values will not reach the optimum within the given epochs at all. SGD is characterized by a good convergence rate for comparable low computational cost.

$$g_t = \nabla_w Q(\mathbf{z}_t, \mathbf{w}_t) \quad (3.11)$$

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta g_t \quad (3.12)$$

To overcome the problem of a fixed learning rate several adaptations of SGD were published. According to Ruder [78] RMSprop, an adaptive learning rate optimization approach, is often used in more complex ANNs. The documentation of the Keras framework [55], the for this thesis used ANN framework, states that RMSprop is suitable also for Recurrent Neural Networks, a special kind of ANNs that will be discussed in detail in a later section. RMSprop was proposed in an introduction lecture about neural networks and machine learning by Hinton [45]. The learning rate is adapted by an exponentially decaying average of squared gradients, which is described in the recursive Equation 3.13. g is the gradient as defined in Equation 3.11. γ is a factor that weights the previous average and the current squared gradient, which is like a momentum that takes the gradient partly further to its previous direction. Equation 3.14 shows the weight update according to RMSprop, after each training example is presented to the ANN. η is the learning rate, as described for SGD and ϵ is a small constant, to avoid division by 0. The by Hinton recommended value for γ is 0.9 and 0.001 for η .

$$E[g^2]_t = \gamma E[g^2]_{t-1} + (1 - \gamma)g_t^2 \quad (3.13)$$

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \frac{\eta}{\sqrt{E[g^2]_t + \epsilon}} \quad (3.14)$$

Adam is another optimization approach, heavily used for complex ANN, proposed by Kingma and Ba [56]. It also makes use of an adaptive learning rate. The adaptive decay rates m_t and v_t are defined in Equations 3.15 and 3.16. Kingma and Ba note that they are biased towards zero during the initial time steps and when the decay rates become small. Because of this they correct these biases by Equation 3.17 and 3.18. The weight update for Adam

is computed as defined in Equation 3.19. The default values, proposed by the authors, are 0.9 for β_1 , 0.999 for β_2 and 10^{-8} for ϵ . Experimental evaluation of the approach showed good convergence results, also for non-stationary problems, which makes this optimization approach a good choice for time series problems solved by ANNs.

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t \quad (3.15)$$

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2 \quad (3.16)$$

$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t} \quad (3.17)$$

$$\hat{v}_t = \frac{v_t}{1 - \beta_2^t} \quad (3.18)$$

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t \quad (3.19)$$

Weight Initialization

To achieve good optimization results the initialization of the parameters at the beginning of the training is an important task, according to Schmidhuber [80]. As of Palit and Popovic [73] a simple random optimization does not always lead to good results. Often an approach, inspired by convex combination methods is applied, where each weight of a weight-vector is initialized by $1/\sqrt{n}$, where n is the dimension of the vector. Glorot and Bengio [38] proposed an initialization approach that draws samples from a uniform distribution as described in equation 3.20, where w_{in} and w_{out} are the dimension of the input- and output-weight-vector. They found their approach to lead to faster convergence and better results at all, especially if activation functions similar to ReLU are used. If not stated other, this initialization approach will be used.

$$\mathcal{U}\left(-\sqrt{\frac{6}{w_{in} - w_{out}}}, \sqrt{\frac{6}{w_{in} - w_{out}}}\right) \quad (3.20)$$

3.3.2. Artificial Neural Network Literature Review

In the following some works that make use of ANN for time series prediction are discussed to underline the importance of this approach. Karunasinghe and Liong [53] used an ANN, in particular a MLP, for prediction of non-linear time series. They evaluated the models on synthetic and real world

chaotic time series. Because of the non-linearity of these data series this is a challenging task. The MLP was found to have a very satisfying forecast accuracy. The models were further evaluated, after adding noise to the test data. This resulted in decreased forecast accuracy but still good results.

Gutierrez et al. [41] used an ANN for forecasting of lumpy spare part demands in 2008. According to the authors this was the first time this kind of model was applied to lumpy spare part demand forecasting. They used a 3-layer MLP model. As input the current demand and the period between the last two successive demand occurrences were taken. Gutierrez et al. compared the performance of the ANN with the classical demand forecasting approaches: Crostons Method, Simple Exponential Smoothing and Syntetos-Boylan Approximation. Despite the simple topology of the neural network it was found to outperform the other models.

Han and Wang used a 4-layer Multi Layer Perceptron for forecasting of multivariate chaotic time series. As preprocessing steps they used phase space reconstruction based on Takens Theorem [91] to find strange attractors, describing the time series underlying dynamical system in a higher dimensional space and Principal Component Analysis, a statistical transformation to exclude correlated features to shrink the dimension of the input data. The model was evaluated on several synthetic and real life data sets, performing with a satisfying overall forecast accuracy.

Ak et al. [1] applied a hybrid model composed of an ANN and multi-objective genetic algorithms to the problem of wind speed forecasting. The parameters of the neural network are optimized by NSGA-II [24], a multi-objective genetic optimization algorithm following the concepts of Pareto optimality and dominance to find a parameter set that is optimal according to several conditions regarding several objectives. Experimental evaluation on real world wind data sets according to different optimization approaches showed NSGA-II to be the best choice. The overall accuracy of the prediction was very high for short term horizon predictions.

In 2013 Zhang et al. [102] proposed a Radial Basis Function neural network model for forecasting of sensor data of an E-Nose. A RBF neural network is a special type of Artificial Neural Network, which makes use of radial basis functions as activation functions. As preprocessing step phase space reconstruction according to Takens embedding theorem [91] was applied. The model was evaluated on collected sensor data and obtained satisfying prediction results,

also for the long-term predictions. Unfortunately, a comparison against other models is missing.

Jaipuria and Mahapatra [51] used a hybrid model composed of a wavelet transformation component and an ANN. The time series is transformed according to discrete wavelet transformation and passed to the ANN to learn the underlying pattern of the data. The proposed model was evaluated on different demand time series and compared against traditional statistical demand forecasting approaches. The hybrid model outperformed the statistical models. It was also found that the ANN approach reduces the bullwhip effect, which describes the amplification of demand noise as demand progresses up its supply chain.

Lolli et al. [65] published a paper about ANN models for the prediction of intermittent spare part demand. Different neural networks were tested with several inputs and hyperparameters, and compared with Crostons Method and SBA. In an expensive statistical evaluation it is showed that the ANN models outperform Crostons Method and SBA. Despite the fact that the Recurrent Neural Network (RNN) was not the best performing model, Lolli et al. noted that the model improves its performance, compared to the other ANN models if the forecast horizon is increased. Among other things, this fact and the well-fitting properties of Recurrent Neural Network models for time series data they are covered in detail in the next section.

3.3.3. Fundamentals of Recurrent Neural Networks

In their book Palit and Popovic [73] describe that the need of networks that can produce a time dependent, non-linear input-output mapping motivated the research of Recurrent Neural Network models. These specialized types of neural networks add the time dimension to its topology and thus introduce memory features to the neural network. One of the first popular recurrent network topology was published in 1990 by Elman [31]. The exemplary structure of the Elman RNN is shown in Figure 3.4. Elman extended the network by a context-layer, which is fed by the hidden layer. The output of the context layer is passed back to the hidden layer in the next time step. Thus, he introduced a one-step delay unit, also referred to as local feedback path. According to Palit and Popovic [73], recurrent networks introduce a kind of loop to the input processing through the network and thus add complexity to the

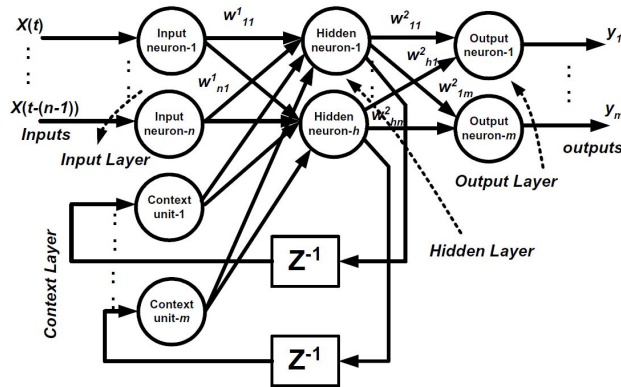


Figure 3.4.: Model of a Recurrent Neural Network: Elman Network [73].

network, which also results in the capability to detect time dependent patterns that could not be detected by basic feed forward networks, like a MLP.

Backpropagation Through Time

The learning of RNNs is done by Backpropagation Through Time (BPTT). The idea of this approach was proposed by several authors, among others by Werbos [97]. BPTT basically works like the basic Backpropagation approach. In difference, to deal with the recurrent layers of the ANN, these layers are unfolded for each iteration of training. The network is trained as a feed forward network with an additional (hidden) layer each training iteration, originating from the recurrent time component. With increasing training iterations, the ANN gets more complex, or deeper, by an increased number of layers. Gradient based training in deep networks arises the vanishing or exploding gradient problem, as stated by Goodfellow et al. [40]. By unfolding the network for too many time steps the gradients for some weights get too small or large and take the optimization in a wrong direction. This led to the development of recurrent units that can solve this problem.

Long Short Term Memory

The Long Short Term Memory (LSTM) is one of these recurrent units that is solving the vanishing or exploding gradients problem. This approach was proposed by Hochreiter and Schmidhuber [46] in 1997. The LSTM has the

ability to model long-term dependencies and short-term dependencies within one unit. It learns what data is stored for how long, as well when and how this data is updated. This is realized by so called gated units within the LSTM unit. A graphical representation of a LSTM unit can be found in Figure 3.5. The LSTM unit is composed of an input-layer, a memory block and an output-layer. The memory block contains adaptive multiplicative gate units to control the information flow, self-connections for modeling the recurrent behavior, as well input- and output-gates for activation of the memory block. The primary unit of the memory block is the Constant Error Carousel (CEC). The CEC processes the information flow within the memory block and represents the state of the LSTM unit. It controls the gated units and therefore manages, which input is processed, when the state of the memory block is reset by the forget-gate and which information is forwarded to the output-layer. According to Hochreiter and Schmidhuber [46], the CEC keeps the network error constant and therefore solves the vanishing or exploding gradients problem. The data is processed through the LSTM by the following equations:

$$g(x) = \frac{4}{1 + \exp^{-x}} - 2 \quad (3.21)$$

$$h(x) = \frac{2}{1 + \exp^{-x}} - 1 \quad (3.22)$$

$$i_t = \sigma(W_{ix}x_t + W_{im}m_{t-1} + W_{ic}c_{t-1} + b_i) \quad (3.23)$$

$$f_t = \sigma(W_{fx}x_t + W_{fm}m_{t-1} + W_{fc}c_{t-1} + b_f) \quad (3.24)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot g(W_{cx}x_t + W_{cm}m_{t-1} + b_c) \quad (3.25)$$

$$o_t = \sigma(W_{ox}x_t + W_{om}m_{t-1} + W_{oc}c_t + b_o) \quad (3.26)$$

$$m_t = o_t \odot h(c_t) \quad (3.27)$$

$$y_t = W_{ym}m_t + b_y \quad (3.28)$$

i_t, o_t, f_t represent the output of the input-gate, the output-gate and the forget-gate. c_t is the activation vector for each cell and m_t the output of the memory block respectively. W and b are the weight matrices and bias vectors of the LSTM unit, connecting all components. \odot represents the scalar product of two vectors and σ is an activation function. The final output of the LSTM unit is computed according to Equation 3.28. Learning of the LSTM unit is done by truncated Backpropagation Through Time, a modified version of BPTT, where the update is performed only every k time steps and backwards only for a fixed number of time steps.

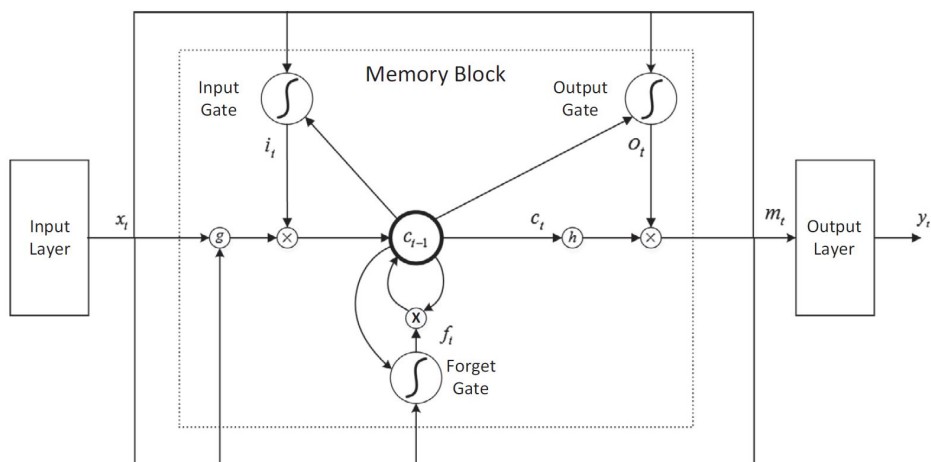


Figure 3.5.: Long Short Term Memory unit [68].

3.3.4. Recurrent Neural Network Literature Review

Because of their specialty to capture time dependent patterns, Recurrent Neural Network models have been heavily used for time series forecasting. An introductory overview, also about other recurrent network types, which exceed the scope of this thesis can be found in the work of Bianchi et al. [6]. In the following a few selected recent works are discussed. Besides the overview Bianchi et al. [6] also performed a comparative study and evaluated several recurrent networks, including Elman RNN, LSTM, Gated Recurrent Units, Non-linear Autoregressive Exogenous model and Echo State Network on synthetic and real world data sets. They found that there is no general solution and that each task has specific requirements to the model. They also found Elman RNN to outperform the more complex gated RNNs, like LSTM on some time series problems, whereas the LSTM outperformed the other tested networks in case the time series is non-linear.

Smith and Jin [84] applied RNN for chaotic time series prediction. They used a multi-objective evolutionary optimization algorithm to train an ensemble of Elman RNN. The proposed model was evaluated on the Mackey-Glass time series and the Sunspot data set, both containing highly non-linear patterns. They achieved satisfactory forecast results with their approach for these problems, that are difficult to predict.

Chitsaz et al. [19] used a RNN for short term electricity load forecasting. The proposed model extracts wavelets, transformations of the data, from the time

series and uses these as inputs for a RNN. Experimental evaluation showed that the recurrent approach is superior to feed forward ANNs supplemented by wavelet transformations, which underlines the utility of RNNs for time dependent prediction tasks.

Chandra [15] proposed a RNN model, supplemented by a competitive cooperative co-evolution optimization, a nature inspired optimization approach. The proposed model was evaluated on several chaotic time series. An extensive comparison against several models from literature showed that recurrent ANNs are superior to the other evaluated approaches. The non-linearity of the chaotic time series was captured with higher accuracy, which resulted in a better forecast accuracy, compared to models like MLP or RBF neural networks.

Gers et al. [37] applied a LSTM model for prediction of non-linear time series data. They tried to evaluate, when to use complex approaches like LSTM compared to simple feed forward networks like MLP. Evaluation on several time series data showed that the LSTM model should be applied only if a simpler approach fails to capture the structure of the data with satisfying accuracy. Furthermore, they propose to combine the LSTM with simpler structures if needed but did not evaluate this proposal.

Ma et al. [68] published a LSTM model for traffic speed prediction. The model was evaluated on travel speed data collected by sensors on an expressway in Beijing. An extensive evaluation against other recurrent ANNs, Support Vector Regression and classical statistical models was done and the LSTM was found to outperform the other models in terms of accuracy and stability. The authors conclude that this underlines the ability of the LSTM to capture characteristics of the time series, like seasonality and trend.

In 2017 Hsu [47] proposed a LSTM model augmented by an autoencoder. An autoencoder is a special ANN that is used for data extraction to get a compressed representation of this. Hsu argues that the LSTM can capture the long-term dependencies of the time series, but has difficulties to capture short-term relations correctly, which he tries to overcome by combining the LSTM with an autoencoder. Experimental evaluation on four data sets, including chaotic time series, shows that the proposed model is superior to other state of the art time series prediction approaches.

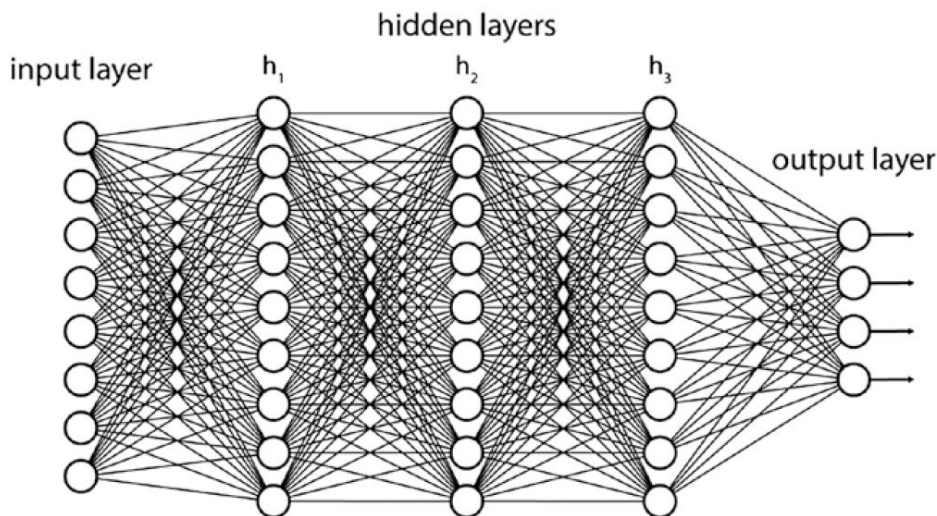


Figure 3.6.: Deep Artificial Neural Network [92]

3.3.5. Deep Learning for Time Series Forecasting

In recent years more and more complex ANN gained research interest and steadily obtained better results. If the (unfolded) graph of the neural network gets deep, which means that it has many layers, it is called Deep Learning, as stated by Schmidhuber [80] and Goodfellow et al. [40]. The number of layers is also referred to as the depth of the ANN. In literature it is not clearly defined, how many layers a neural network at least needs, to call it deep. For this work networks with at least three hidden layers are regarded as deep. RNN can be regarded as deep by its nature because unfolding of recurrent units adds automatically depth to the unfolded network graph with each processed timestep. An exemplary graph of a deep ANN is shown in Figure 3.6. The structure of the network, defined the depth, width and types of layers, is called the network architecture or topology. Taweh [92] describes that each layer of a deep network learns a level of abstraction of the given data until the desired complexity of the representation is reached. Mathematically the data is transferred from one space to another by each layer until the solution space is reached.

Busseti et al. [12] proposed a deep ANN for electricity load prediction. They compared deep feed forward networks with deep RNN and other state of the art models on real world data sets of the electricity sector. The deep RNN was found to be superior according to the forecast accuracy. The authors also

state, that the performance of the deep ANN highly depends on the network topology. They showed that deep architectures can deal with the non-linearity and seasonality of the electricity load time series.

Kuremoto et al. [58] published a deep model composed of several layers of Restricted Boltzmann machines, a special type of neural network. They used a combination of Backpropagation and Particle Swarm Optimization to train the ANN. The proposed model was evaluated on the CATS benchmark data sets [61] and several chaotic time series. Evaluation showed the superiority of the deep model compared to simpler ANN.

Yeo [100] applied a deep LSTM model to chaotic time series data. The output layer was modified to return a confidence interval instead of precise forecasts. Experimental evaluation on several synthetic chaotic and real world data sets showed the proposed model to reach a satisfying forecast accuracy, even if the data is highly non-linear. Yeo concludes that deep models are a powerful tool for prediction of dynamical systems.

These are only a few selected examples of deep ANN in the area of time series forecasting. Interested readers are referred to the work of Laengkvist et al. [64] and the paper of Gamboa [35] for an introductory overview. Both mentioned surveys conclude that deep learning is an emerging approach with promising results (also) in the area of time series prediction.

To the authors best knowledge there is currently no published work, applying deep learning techniques beyond RNNs for spare part demand forecasting.

4. Data Basis and Current Model

This chapter provides a description of the data, its features and briefly summarizes the data preparation steps. Furthermore, the current modeling approach is discussed and analyzed. Possible enhancements of the current model that could improve the forecast accuracy are proposed and reviewed.

4.1. Spare Part Demand Data

The real world data used for this thesis is provided by a large, worldwide operating, automotive OEM. The data contains plenty features and several additional derived features. For the scope of the model to be developed only a selection of the provided data is needed. This works focus is set on young fast-moving spare parts with or without Vehicle Production Data (VPD). A part is regarded as fast-moving if the Average Demand Interval (ADI), the average of all intervals between two successive demands is less than 1.51 months. A part is considered as young part if the last month with demand is within the current year and the period between the first demand occurrence and the last demand occurrence, the demand period, is within the interval of 12 to 59 months. Furthermore, an average monthly demand greater than 7 is taken into account as selection condition. Table 4.1 summarizes the selection criteria of the parts that are covered by the model developed within this work.

For evaluation sufficient historic demand data for each part is needed. Thus, the data also contains parts that fulfilled the selection criteria in the past and now provide demand data for a longer period. In total data of 7191 different parts with VPD and 4989 parts without VPD is available. The data ranges from January 2007 until December 2017. Figure 4.1 shows the distribution of demand intervals, the range from first until last demand occurrence per part,

	STPM-VPD	STPM
Average Demand Interval	< 1.51	< 1.51
Demand period in months	$12 \leq t \leq 59$	$12 \leq t \leq 59$
Last demand occurrence	within current year	within current year
Average monthly demand	> 7	> 7
Vehicle Production Data	available	not available
Number of parts	7191	4989

Table 4.1.: Criteria for data selection.

for all 12180 different parts contained in the data. Most parts have a history larger than 60 months. This is useful for the evaluation process. For evaluation of the models a hold-out-sample will be used. The model will be trained on a fixed period, namely the first 24 months after the first demand occurrence and evaluation is done on the complete period until the last demand occurrence. This evaluates on the one hand side how well the model could fit the training data and on the other hand side how accurate the future is predicted by the model, which is evaluated by the remaining historic demand data, not used for model training, the so-called hold-out-sample.

Data pre-processing steps, e.g. outlier detection and removal, are done by IBM before the data is passed to the model. In the scope of this work only the cleaned data is used, so a detailed description of the pre-processing and data cleansing steps is abandoned.

The data available for this thesis contains the following features:

- An explicitly identifying **part-number** (anonymized due to data privacy constraints)
- The **month**, as a continuous number composed of year and month in the format *YYYYMM*
- The historic **demand** of each month as integer
- The historic and future **vehicle production** of each month as integer (optional)

The data was aggregated on a monthly basis. As provided by the OEM the historic demand is on a daily level. Tests, performed by IBM showed that the current model can handle data best if the demand is aggregated monthly.

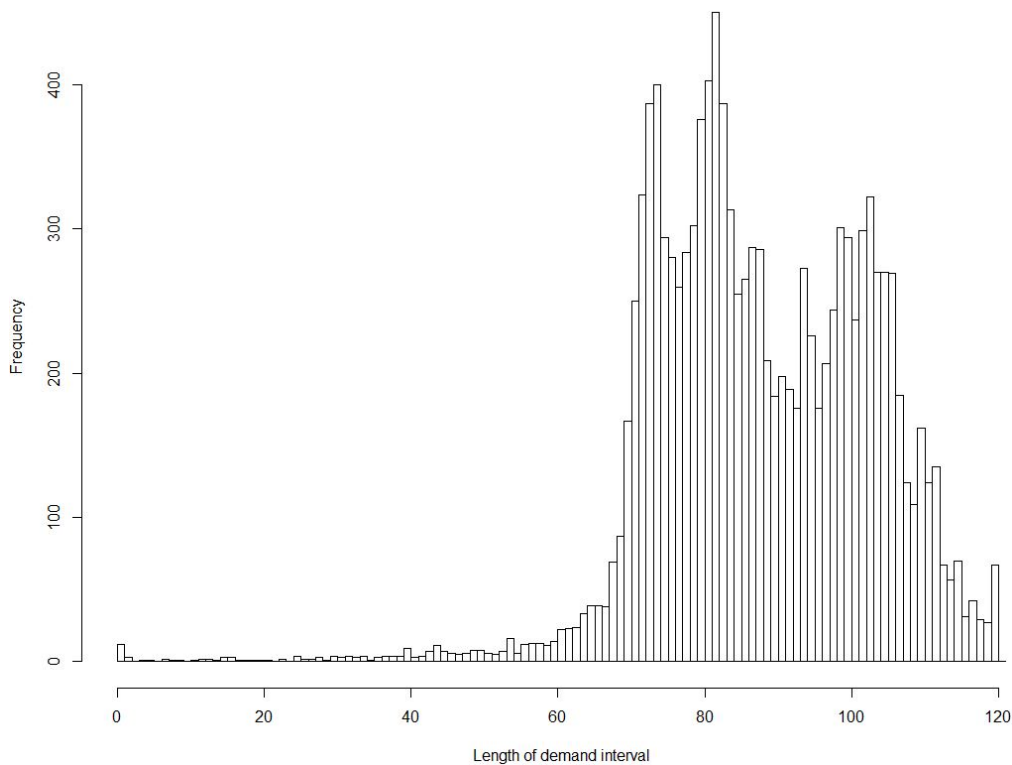


Figure 4.1.: Histogram: Demand period per part.

Aggregated by months the time series becomes smoother and the non-linearity is decreased, which results in data, easier to forecast. Figure 4.2 shows the demand for an exemplary part over the same period. The abscissa shows time and the ordinate shows demand. The data is aggregated on a daily, weekly and monthly level. The daily data is intermittent, which means that there are periods with no demand at all, with a broad spectrum, which is indicated by plenty peaks in the demand curve. The weekly data only has a broad spectrum, but the demand curve is already smoother than for the daily data. The weekly data also rarely has periods without demand. The monthly data has a less wider spectrum and usually no periods without demand at all. It could be assumed that the order process of the parts, based on the exact dates, is performed on a monthly basis. This results in a strong seasonality within a month, which is removed if aggregated to months. The demand curve usually gets smoother the higher the aggregation level becomes. A higher level than monthly aggregation is dismissed because the data points available for training the model get too low. The Vehicle Production Data is provided on a yearly

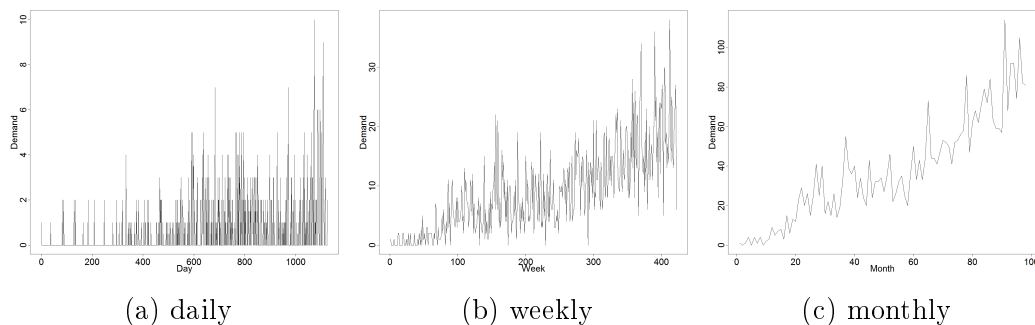


Figure 4.2.: Same period of demand on different aggregation levels.

basis. Because of the monthly aggregation level the VPD is equally distributed over all months of a year.

4.2. Current Model

The Short-Term Prediction Model, short-term representing the short period of historic demand data available for model training, is based on a regression approach. It exists a model for parts with VPD, taking the multivariate time series as input and a model for parts without VPD, using the univariate data respectively.

4.2.1. STPM-VPD Model

The STPM-VPD model takes the historic demand and the VPD as multivariate time series input. Based on six different parameters a regression model is build to forecast the spare part demand. The parameters are

- α_f as part failure rate,
- α_d as decay / increase rate of the part failure rate,
- α_o as offset, when part failures start to affect the demand,
- β_f as vehicle depletion rate,
- β_d as decay / increase rate of the vehicle depletion rate,
- β_o as offset, when vehicles start to disappear from market.

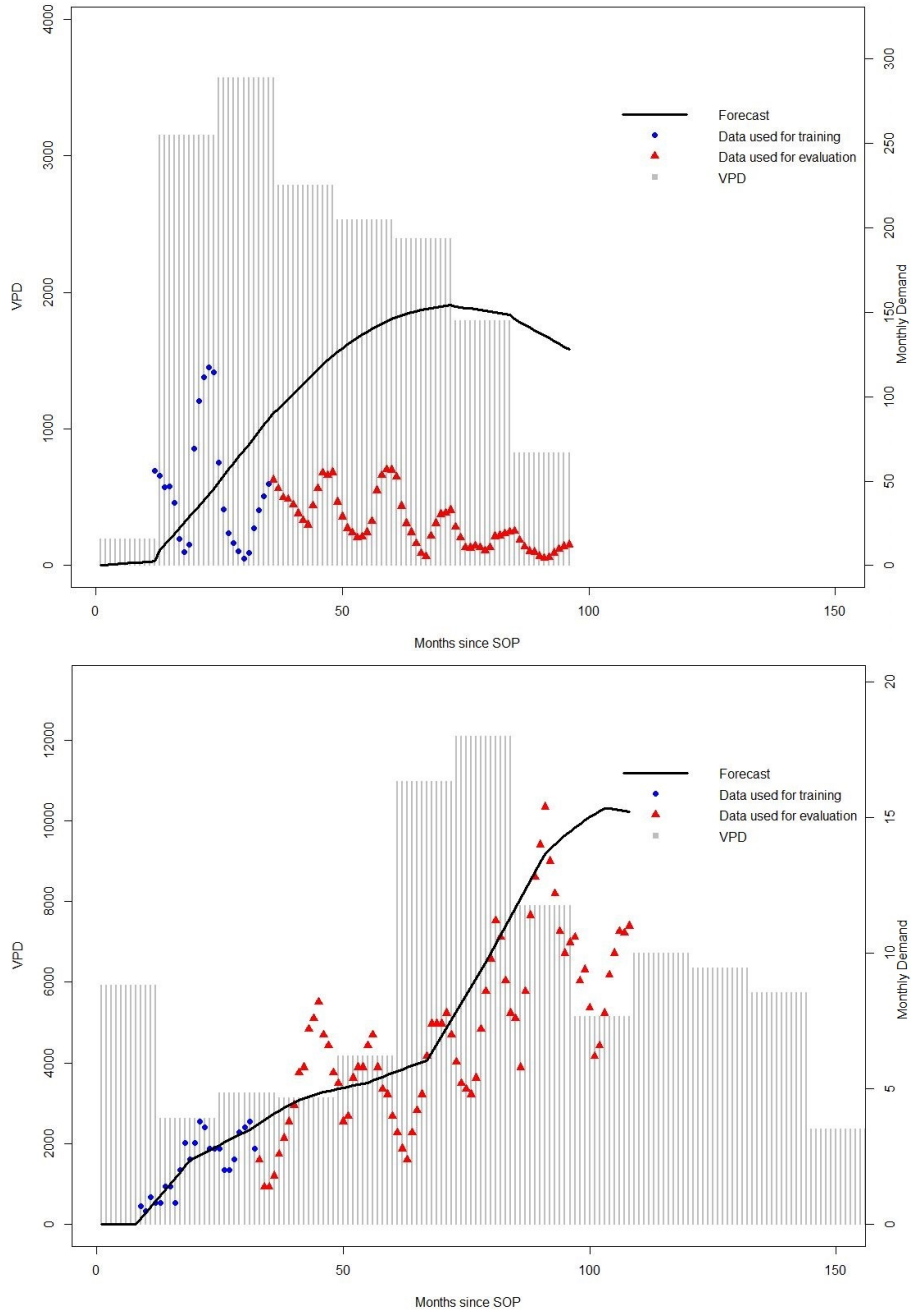


Figure 4.3.: Examples for STPM-VPD predictions.

Based on the training data these parameters are initially guessed. The forecast is calculated based on all six parameters, weighting the cumulative amount of in the market remaining vehicles for each time-step, the guessed part failure rates and the historic demand. According to a one-dimensional optimization the vehicle depletion rate β_f is systematically tweaked to minimize the error between the prediction, based on the current parameter set and the true historic demand. The final prediction is then calculated according to the optimized vehicle depletion rate. Figure 4.3 shows the prediction of the STPM-VPD model exemplary for two different spare parts with VPD. The spare part demand is represented on the right ordinate and the VPD is shown with different scale on the left. The first diagram shows a rather overestimated demand prediction, whereas the second one visualizes a forecast that very accurately captures the structure of the demand.

4.2.2. STPM Model

The STPM model takes the historic spare part demand as univariate time series input. Based on a linear regression approach two trend parameters α_t and β_t are derived for each time step of the historic demand. According to the two parameters a first model is fitted to the training data. The pre-processed demand data is supplemented by a demand of 0 at the guessed End of Life of the part to force the model to a prediction, decreasing to zero until the end of the prediction horizon. In a second step this time series then is used as input for a cubic spline interpolation model. The cubic spline model is finally applied to forecast the values in between the end of the demand history used for model training and the guessed EOL. Some exemplary predictions of the STPM model are shown in Figure 4.4. The diagram of the first part shows a prediction overestimating the demand. The second plot shows a satisfactory prediction.

4.3. Enhancements of Current Model

One of the targets of this work is to evaluate, whether the forecast accuracy of the current model could be improved. Based on an analysis of the current model flaws have been identified. The following sections describe some of

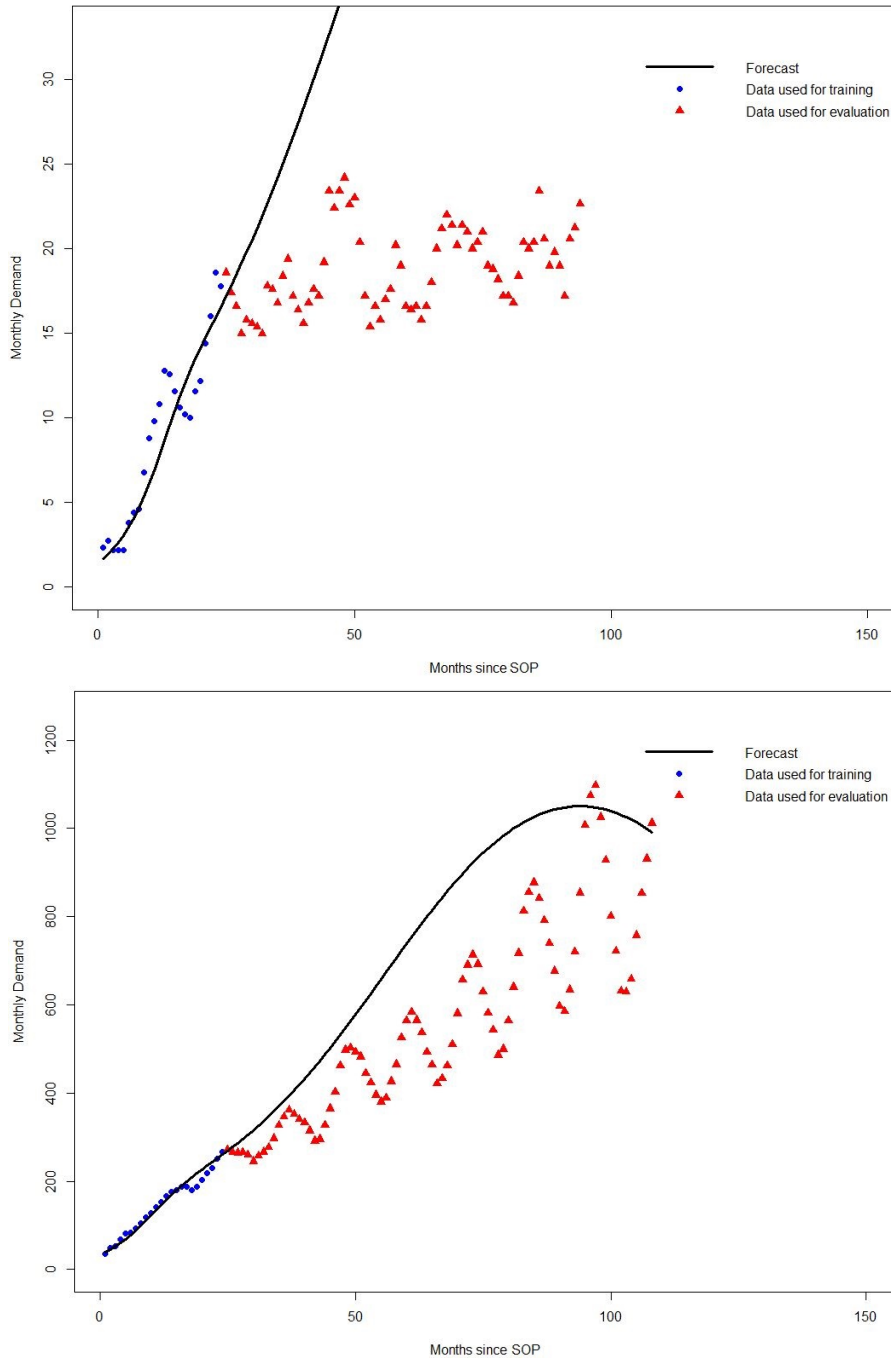


Figure 4.4.: Examples for STPM predictions.

these weaknesses of the current models and try to overcome them by proposing improvements that shall increase the forecast accuracy of the approaches. Each of these improvements is described, the performance of the enhancements is compared against the currently applied model and the outcome is discussed.

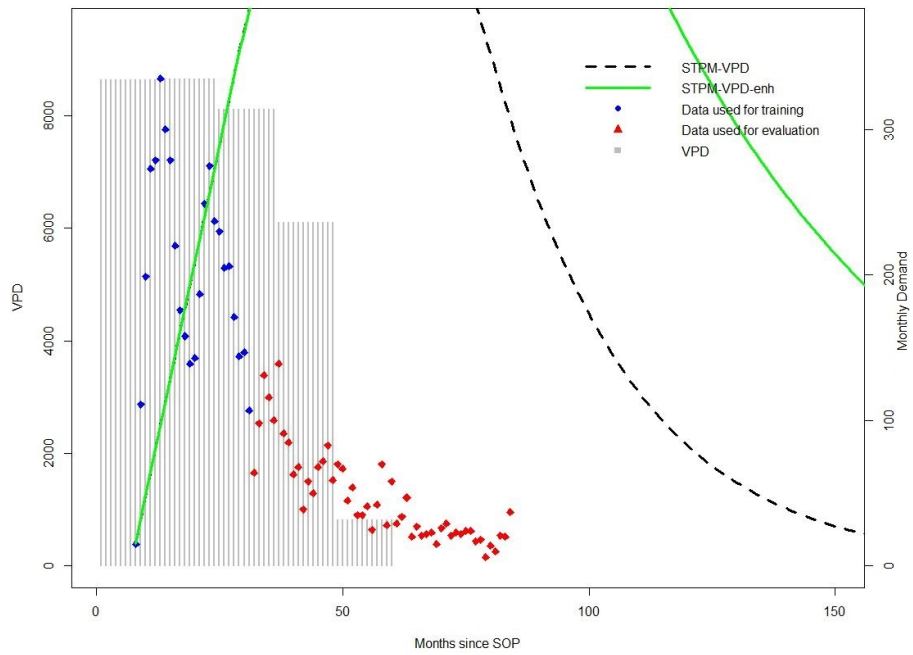
4.3.1. Enhancements of STPM-VPD Model

One of the weaknesses of the STPM-VPD model is that only one of its parameters is optimized, the others are guessed. This motivated the idea to apply a constrained based multi objective optimization approach, which involves all six parameters. The constraints were defined based on previous STPM-VPD experiences, to shrink the solution space. For optimization a Downhill-Simplex approach [72] was applied. The models were evaluated on a random sample of 40 parts, which according to IBM showed good generalization properties in past experiments. The models are trained on the first 24 months of demand history and evaluated on the complete available data. The forecast accuracy was rated according to Chi-Squared-Distance as defined in Equation 4.1. x_t and y_t are the historic and the predicted demand at time t , each normalized by their overall sum. T is the total number of time-steps.

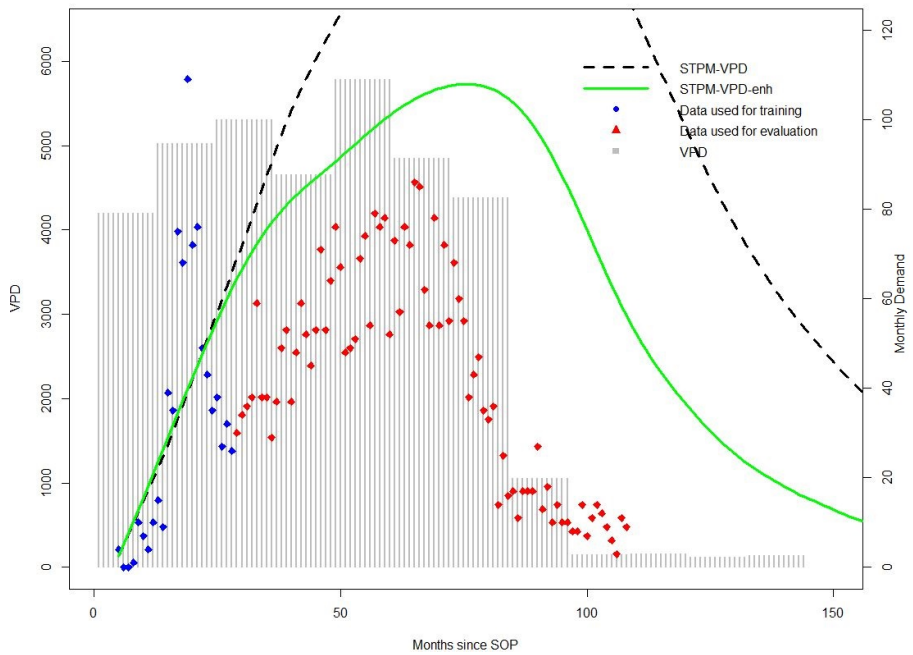
$$\chi^2 = \frac{1}{2} \sum_{t=1}^T \frac{(x_t - y_t)^2}{(x_t + y_t)} \quad (4.1)$$

For evaluation the results of a version of the currently applied model with and without the enhancement are run on the same sample and the forecasts for each part are compared according to their Chi-Squared-Distance to the historic demand. Optimization of all six parameters led to an increased forecast accuracy according to Chi-Squared-Distance for 45% of the parts of the tested sample. If only α_d , β_f , β_d and β_o are optimized and the other parameters are guessed, as before, the forecast accuracy according to Chi-Squared Distance is increased for 52.5% of the parts. Due to the increased computational complexity the improvement of the forecast accuracy is a rather small benefit. It may be noted that different optimization approaches, e.g. Differential Evolution [87] performed worse, than the applied algorithm.

Another enhancement tries to overcome the assumption that the VPD is equally distributed over a year. To smooth the VPD a polynomial is fit to the data. A polynomial of degree 15 was found to perform best according



(a) no enhancement



(b) enhancement

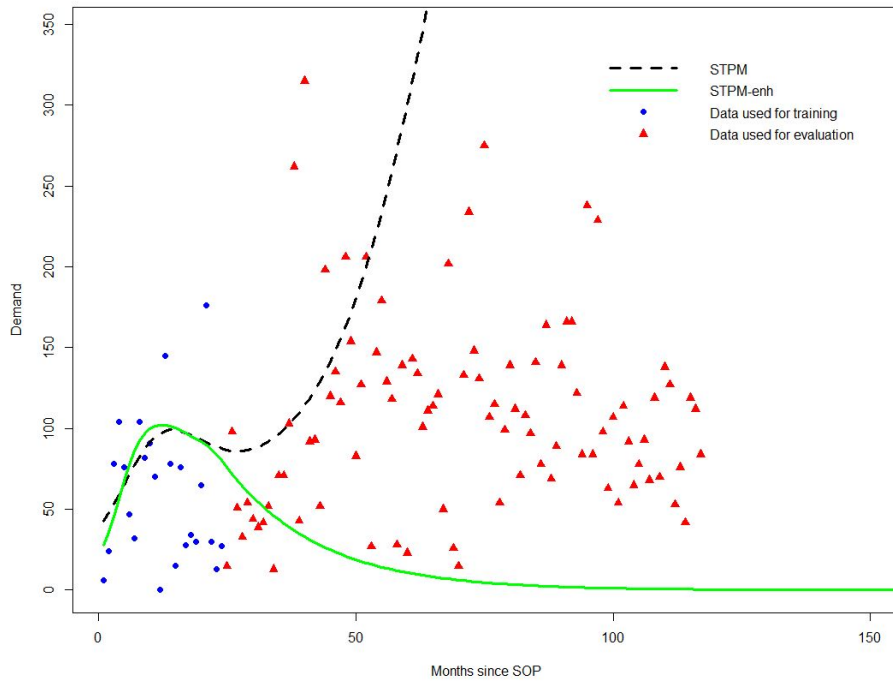
Figure 4.5.: Comparison STPM-VPD versus STPM-VPD-enh predictions.

to the forecast accuracy of the STPM-VPD model. The model with polynomial smoothed VPD input increased the forecast performance for 60.5% of the parts, nevertheless the enhancements were rather small compared to the overall accuracy. It may be noted, that the smoothing by a polynomial does not keep the original sum of vehicles per year. Prototypical tests with another linearization technique that keeps the original sum of vehicles per year showed an improvement of prediction accuracy for only 37% of parts. This concludes, that it seems to be more important to smooth the input data than keeping the actual sum.

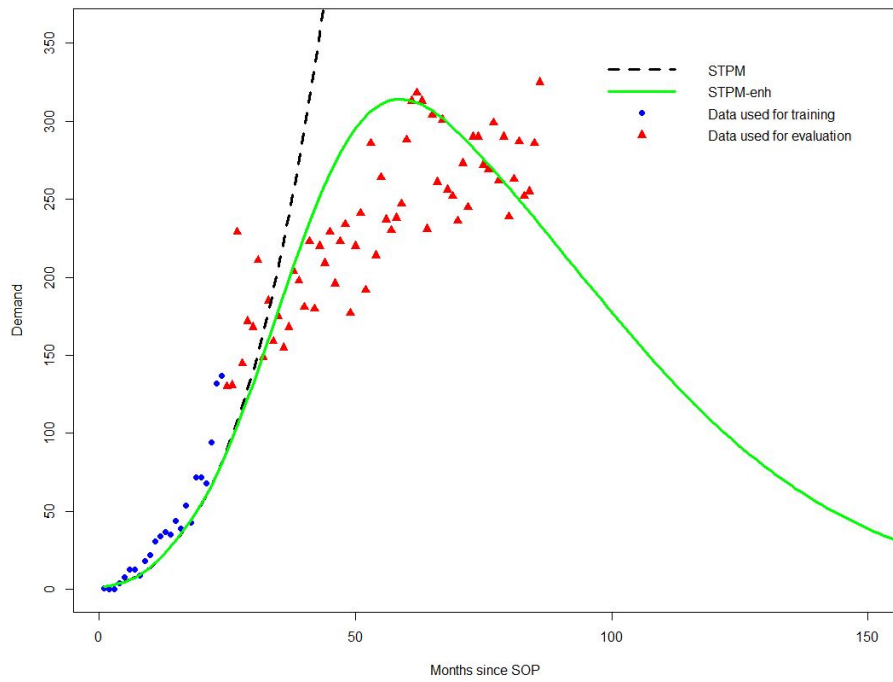
Figure 4.5 presents two plots of the predictions of the enhanced, denoted as STPM-VPD-enh and the basic STPM-VPD model on exemplary spare parts. The accuracy of the forecast of diagram 4.5a was further decreased by the proposed enhancements, even stronger overestimating the spare part demand. Plot 4.5b features a prediction that is boosted in terms of accuracy by the improvements to the multivariate time series model, unfortunately still slightly overestimating the demand. Concluding, these are only a few possible enhancements of the STPM-VPD model. To cover all possibilities, e.g. forecast plausibility checks or additional parameters, would exceed the scope of this work. Summarizing it can be stated that all improvements have a rather small influence on the forecast performance compared to the real spare part demand. Furthermore, the changes to the model influence each other and the effects do not always sum up positive. Therefore, increasing the model performance is possible but becomes a tough and extensive task, which benefits the idea of a fundamental different approach.

4.3.2. Enhancements of STPM Model

The STPM model overestimates the demand for plenty of parts. This is caused by the limited amount of data and the amplification of a demand growth in the first months of demand history. To overcome this overestimation a forecast plausibility check is added to the model. As benchmarks the slope of a straight linear curve of the first few predicted months and the relation between the average historic demand and the average predicted demand are applied. Rules with estimated threshold values check whether the prediction is plausible or not. In latter case a down-scaling is performed. Therefore, a guessed value, a multiple of the average historic demand, is assumed at the point in time with



(a) no enhancement



(b) enhancement

Figure 4.6.: Comparison STPM versus STPM-enh predictions.

the highest forecast value and the STPM model is calculated again according to the historic data supplemented with the assumed demand value.

An enhanced version and the currently applied model are compared on a sample of 40 spare parts, selected by IBM based on previous experimental experiences. Like for the model with VPD, the first 24 months of demand history are used for model training and the complete historic demand data is used for evaluation. The plausibility check of the prediction led to an improvement of forecast accuracy according to Chi-Squared-Distance for 78% of the tested parts compared to the currently applied model. Nevertheless, the scaled predictions still often overestimate the real demand. The derivation of the rule threshold values is an expensive task that gets even more complex, if the number of different rules applied grows.

Figure 4.6 shows a comparison of the enhanced univariate time series model, denoted as STPM-enh with the basic STPM model on some exemplary parts. The plot of 4.6a shows a case where neither the enhanced version, nor the basic version satisfactory predicted the spare part demand. The diagram in 4.6b represents a part, the forecast is improved by the proposed enhancement compared to the basic model. Nevertheless, due to the limited information, fitting the STPM model is a tough task. Even if the plausibility check improved the forecast performance for plenty of parts the overall accuracy related to the real demand is still not satisfying. Because of the overall prediction accuracy and the limited possibilities to tune the model the current approach should be questioned at all.

5. Deep Learning based Approach for Spare Part Demand Forecasting

Based on the theoretical foundations and literature review from Chapter 2 and 3 this section introduces a deep learning based model for spare part demand forecasting. The current model has some weaknesses, as described in Chapter 4, that should be dealt with by a fundamentally different approach. First the new approach is introduced. Then the hyperparameters of the model are derived and statistically evaluated. Finally, the findings of this chapter are summarized and discussed.

5.1. Deep Learning based Model

Time series are characterized by features like linearity, trend, seasonality and stationarity. All these features require a model that is capable of representing these properties. Based on the literature review from Chapter 3 Support Vector Regression and Artificial Neural Network models are the two most promising approaches, recently often applied in research, that are capable of dealing with these features. Support Vector Regression stands up by the ability to find always an optimal solution. ANNs feature by outstanding pattern recognition capabilities. Both techniques are sensible to hyperparameter configuration. As also seen from literature review, which model is superior depends on the task. There exists no model that outperforms all the others. Because both models are promising alternatives to the current approach a Proof of Concept (POC) test was performed with SVR and ANN for spare part demand forecasting based on the provided data. This should assist as a decision basis, which technique should be evaluated in detail.

The POC showed, that SVR is not competitive compared to the currently applied model, whereas a simple Multi Layer Perceptron was already performing well. One possible explanation of this result is that there is too few training data available for a SVR approach to find an optimal solution. Al-Saba and El-Amin [2] found ANN to perform well, even if there is a low amount of training data. Nonetheless, the POC was not of statistical accuracy. The promising results of the MLP could, for example depend on lucky parameter initialization.

Literature review furthermore showed Recurrent Neural Network models and deep ANNs to perform well on time series forecasting tasks. In case of RNNs this is based on their capability of learning time dependent patterns and in case of deep networks the ability of representing highly non-linear relations within the data can be mentioned. So, the POC was extended by these approaches to get an overview, which models are suitable for the current task and if the limited amount of training data is enough to even train more complex neural networks. ANNs with recurrent and densely connected layers, like the layers of a MLP, showed the most promising results.

To the authors best knowledge, such an approach was not applied to spare part demand forecasting yet, even if some works like the paper of Busseti et al. [12] and the work of Yeo [100] proposed similar models for different time series problems. Lolli et al. [65] applied single hidden layer ANN for spare part demand forecasting, which can be regarded as a work motivating the idea of applying more complex networks, as stated in their outlook. The above-mentioned points led to the decision to detailed evaluate a deep learning based model for the task of spare part demand forecasting.

Deep Learning is characterized by many hyperparameters that could be tuned. According to Busseti et al. [12] the topology of a deep ANN is the most important of these influence factors. For this work hidden layers of three different types of layers are used, as described in Section 3.3.1 and 3.3.3:

- **Densely connected** layers are layers, where each input is connected to each neuron and the output of each unit is forwarded to each neuron of the next layer. The densely connected layer works as an input processing unit, learning patterns and transforming the data in space. An ANN composed of densely connected layers is shown in Figure 3.1 on page 27.
- **Elman layer**, or simple recurrent layer, is a layer, where the output is delayed one time step and used as additional input in the next time step

via a context layer. The Elman layer represents a short-term memory. The structure of an Elman network is shown in Figure 3.4 on page 35.

- **Long Short Term Memory** is a layer, that independently learns what information is stored, for what periods. The LSTM function as a long- and short-term memory layer, storing information that is regarded as important by the deep ANN. Figure 3.5 on page 37 visualizes the structure of a LSTM unit.

The special capabilities of the three types of layers shall learn the time series features from the training data and accurately predict the future by one-step-ahead forecasts. The densely connected layers are regarded as pattern learning units that shall prepare the input for the recurrent layers and learn auto-correlations between the different features of the multivariate time series data. The recurrent layers shall learn time dependent features. The Elman layer is regarded as short-term memory, connecting only to the previous time step. The LSTM is regarded as self-learning memory that independently decides, which information is important for the current time series. The depth and width of the ANN are derived experimentally in a later section. To limit the space of possible topology, building blocks are defined. Each recurrent layer is followed by a densely connected layer to process the output and prepare it for the next recurrent layer. Furthermore, the different possible depths and widths are also limited.

As stated in section 3.3.1, different optimization algorithms could be applied for ANN parameter learning. Stochastic Gradient Descent, as one of the most used optimizer, is applied as a baseline. Furthermore, RMSprop and Adam as specialized optimizer for deep and recurrent ANNs are evaluated. The learning-rate as hyperparameter of the optimizer is derived experimentally in combination with the optimizer and is described in more detail in a related section later. Mean Squared Error (MSE) is applied as error function to optimize. MSE is defined in Equation 5.1, where \mathbf{x} is the historic demand vector, $\hat{\mathbf{x}}$ the predicted demand vector and T the number of time steps, or dimension of the vector.

$$E_{MSE} = \frac{1}{T} \sum_{t=1}^n (x_t - \hat{x}_t)^2 \quad (5.1)$$

As activation functions ReLU, leaky ReLU and SoftPlus, as defined in Section 3.3.1 are applied. According to literature, e.g. by Glorot et al. [39], these

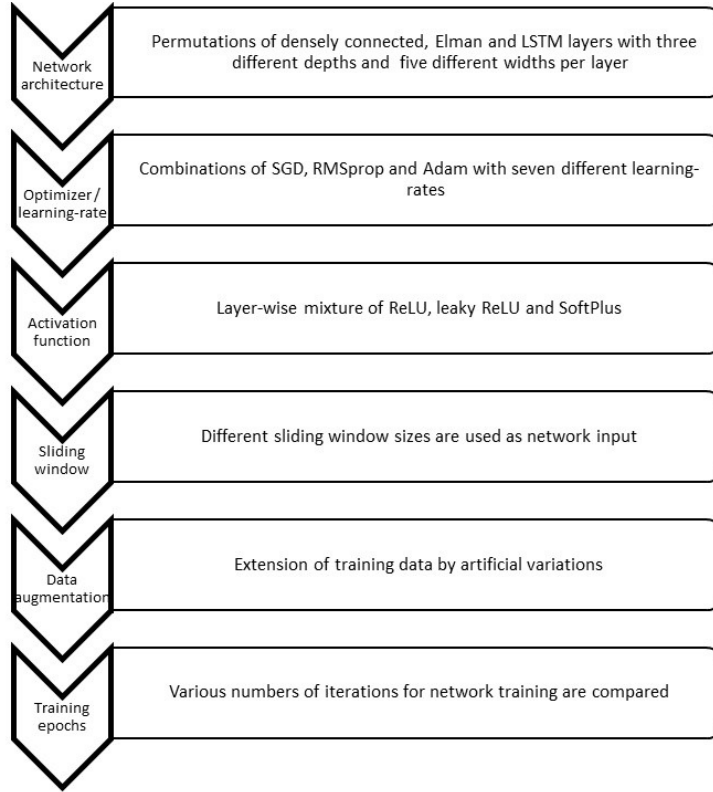


Figure 5.1.: Order of hyperparameter determination.

are the most suitable functions for recurrent and deep ANN. Further hyperparameters considered are the number of training epochs and the size of the sliding window. The sliding window size defines how many past values are used as input for each time step. The window is then moved sequentially through the data. The number of training epochs defines how many training cycles for the given training data are completed until the optimization of the network parameters is finished. Additionally, data augmentation as an input-related optimization process is evaluated. By data augmentation, the training data is extended by artificial variations to evaluate the influence of a larger number of training instances available.

All mentioned hyperparameters are derived by statistical experiments to build a separate deep learning based model for parts with VPD, in the following referred to as DL-STPM-VPD, and for parts without VPD, referred to as DL-STPM. As input for the DL-STPM-VPD model, the historic demand, the VPD and the cumulative VPD at time t are used. Based on the historic demand

and the VPD, the model should learn the relation of both, e.g. part depletion rate. The cumulative VPD shall help the model to determine the remaining cars in the market. Further inputs like the future VPD are omitted based on experiences from the POC. For the DL-STPM model only the historic demand at time t is available as input. For all models the training horizon is fixed at 24 months, meaning the training data contains 24 different time steps. This constraint is assumed because of convenience for evaluation, clearly separating training and validation data.

To derive the hyperparameters the experiments are performed in a sequential process. The results of the completed experiments are used as configuration input for the successive tests. In deep learning literature exists no golden road for hyperparameter determination. Based on recommendations and best practices the order of hyperparameter as seen in Figure 5.1 is followed in the next sections.

5.2. Experimental Setup

The following section describes the framework for the experiments. The used evaluation functions are introduced, and the selection of appropriate spare part samples are discussed. Furthermore, the significance evaluation as major quality measure is introduced.

5.2.1. Evaluation Functions

To evaluate the forecast accuracy of the proposed model different evaluation functions will be used. In the following \mathbf{x} and $\hat{\mathbf{x}}$ represent the historic demand vector and the predicted demand vector, both of same length and T is the dimension of the vector, representing the number of time steps. As main evaluation measure the Root Mean Squared Error (RMSE) is chosen. This scale dependent distance-based error function is widely used in literature. As MSE is used for weight optimization of the ANN, Root Mean Squared Error is preferred for evaluation because it is in the same scale as the data. It is defined in Equation 5.2. Additionally the Chi-Squared-Distance [59], as distance-based error function that has been used by IBM previously in the project and is therefore known by the customer as quality measure, as well as

the Correlation Coefficient (CC) as similarity-based error function are introduced in Equation 5.3 and 5.4 respectively. These shall supplement the results of the main evaluation function RMSE and avoid misleading conclusions by relying on only one evaluation function. For the both first mentioned evaluation functions a smaller value indicates that the prediction is closer to the real demand. The values for the CC are in the interval $[1, -1]$. Values closer to 1 indicate a stronger correlation between the historic and predicted demand, which means that the prediction is similar to the history. Even though there exist plenty other evaluation measures, a review of several functions can be found in the work of Hyndman and Koehler [49], this selection was chosen based on the literature analysis and previous project experiences.

$$E_{RMSE} = \sqrt{\frac{1}{T} \sum_{t=1}^T (x_t - \hat{x}_t)^2} \quad (5.2)$$

$$E_{\chi^2} = \frac{1}{2} \sum_{t=1}^T \frac{(x_t - \hat{x}_t)^2}{(x_t + \hat{x}_t)} \quad (5.3)$$

$$E_{CC} = \frac{T(\sum_{t=1}^T x_t \hat{x}_t) - (\sum_{t=1}^T x_t)(\sum_{t=1}^T \hat{x}_t)}{\sqrt{(T \sum_{t=1}^T x_t^2 - (\sum_{t=1}^T x_t)^2)(T \sum_{t=1}^T \hat{x}_t^2 - (\sum_{t=1}^T \hat{x}_t)^2)}} \quad (5.4)$$

To evaluate, which model performed best on a particular part in an experiment a tournament evaluation is applied. The approach is described in Algorithm 1. The error vectors for each model or configuration on each part, containing the error values of all runs of the model, are calculated according to the defined evaluation functions in Line 5. For tournament evaluation the median error of each error vector is considered, which is extracted in Line 7. For a spare part of the evaluated sample a ranking according to each evaluation function is created in Line 9. A model gets a point for each model it outperformed according to an evaluation function, as defined by Line 10. These points are summed for each model over all evaluation functions, resulting in a final score for that particular part in Row 12. The model with the highest score is regarded as best model for this particular part. This process is performed for all parts of a sample to get the best model for each part. If the score is summed for each model over all parts of a sample, the best model of the sample can be found. This optional step is performed in Line 14. If not stated else, all three above described evaluation measures are taken into account for tournament scoring.

Algorithm 1 Algorithm of tournament evaluation.

```

1: for each p in Spare parts do
2:   for each e in Evaluation functions do
3:     for each m in models do
4:       for each r in Runs of model do
5:          $ErrorVector[e, m, r] \leftarrow e(p, m)$ 
6:       end for
7:        $MedianError[e, m] \leftarrow Median(ErrorVector[e, m])$ 
8:     end for
9:      $Ranking[e] \leftarrow CreateRanking(e, MedianError[e])$ 
10:     $Score[model, p, e] \leftarrow NumberOfModels - RankOfModel$ 
11:  end for
12:   $Score[model, p] \leftarrow \sum_e Score[p, model, e]$ 
13: end for
14:  $Score[model] \leftarrow \sum_p Score[model, p]$ 

```

5.2.2. Sample Selection

For evaluation random samples of the parts are drawn because calculation on all available parts would simply take too much time with available computational resources. For each evaluation step, deriving the deep learning based model, a sample of 40 parts is used. A sample of that size showed good generalization results during other tests performed by IBM with the current data set. The sample-size can be calculated according to the Cochran formula [20], shown in Equation 5.5. Z is the z-score, describing the area under the bell curve of a Gaussian distribution, according to a desired confidence interval, which could be derived from a Standard Gaussian z-Table. For this work we assume a confidence interval of 95%, which results in a z-score of 1.96. p represents the proportion of the desired outcome. As this proportion is unknown, 0.5 is assumed, which is usually taken as value for p if the true proportion of the classes in the sample, in this case parts, where a model is superior to the other model, is unknown. q is $1 - p$ and e represents the margin of error, within which the results should range.

$$n_0 = \frac{Z^2 pq}{e^2} \quad (5.5)$$

According to Cochran's formula a sample size of 40 with a confidence interval of 95% results in a margin of error of 15% for both models. This margin of error

is accepted in favor of the computation time of the experiments, even though it allows wide spread results that could lead to a wrong direction in worst case. Furthermore, the sample size calculation assumes a Gaussian distribution. As the distribution of error is not known to be normally distributed, each experiment need to be repeated several times. As of the central limit theorem [42], a sampled error distribution is normal distributed if a large enough number of independent random samples with replacement are taken from the error distribution. A rule of thumb states that at least 30 samples should be taken. Thus, each experiment will be repeated 31 times. For each experiment a new sample of 40 parts is drawn from the multivariate and univariate time series data for the model with and without VPD respectively. On the one hand side this should avoid overfitting of the model on a particular training sample and on the other hand side better generalization capabilities of the model should be achieved.

5.2.3. Significance Test

To ensure statistical significance of the results and to avoid decisions by coincidence, a significance test will be applied to the experimental outcomes. As significance test the Wilcoxon-Rank-Sum-Test, also known as Mann-Whitney-U test, will be applied. As stated by Walpole et al. [96] this non-parametric significance test has weaker requirements to the compared distributions than for example the paired t-Test. The significance test checks whether a null-hypothesis is correct or not. In our case the null-hypothesis states that both compared error vectors are sampled from the same distribution. The test estimates a p-value. If the p-value is less than or equal to a significance level $\alpha = 0.05$, the null-hypothesis is rejected, which means that both error vectors are drawn from different distributions and the result can be regarded as significant.

To find the best performing model according to the significance test, two significance measures are defined in the following. The *best-model-significance* of a model M , short ψ_{bm} , is defined by the total number of models that performed significantly worse on parts, where M performed best. To determine ψ_{bm} , for each part it is calculated, which model or configuration performed best according to the applied evaluation functions. The error vector, containing the errors of all runs of the best model on this part according to the in the previous

section defined primary evaluation function RMSE, is compared to the error vectors of the other models or configurations of a particular experiment on this part. If the p-value of one of these tests is less than or equal to 0.05 the tested model or configuration is considered as significantly worse than M . Over all parts these significant worse models or configurations are counted and summed up for each best performing model, resulting in a *best-model-significance* value for each model taking part in the experiment. The model or configuration that in the end has the highest ψ_{bm} value is regarded as the significant best model of the particular experiment. The maximal reachable ψ_{bm} score is calculated according to Equation 5.6, where N is the number of models involved in the evaluation process and P is the number of parts, contained in the sample.

$$\psi_{bm-max} = (N - 1)P \quad (5.6)$$

The second significance measure introduced is the *spare-part-significance*, short ψ_{sp} . To determine ψ_{sp} for a model M , for each part of the sample the best performing model as of E_{RMSE} , E_{χ^2} and E_{CC} is identified. For each spare part the RMSE error vector of the for this part best model is taken as reference vector. This reference vector is compared with the error vector of M for the particular part and a p-value for that part is calculated. In case this p-value is less than or equal to 0.05 it is stating that M performed significantly worse than the best model of this part. This process is done for all spare parts of a sample and the number, M performed significantly worse is counted over all parts of a sample, resulting in the ψ_{sp} measure. The *spare-part-significance* indicates, whether a model M produces competitive results on parts of a sample itself were found not to perform best. Smaller values are better, representing a model less often significantly worse than the best model per part. The minimal value is zero in case the model performed never significantly worse than another and the maximum value is equal to the number of parts contained in the sample used for evaluation.

5.3. Hyperparameter Determination

As stated in section 5.1, the proposed model contains plenty of hyperparameters that could be configured and tuned to achieve an optimal result. This section will describe the experiments that have been performed to determine the hyperparameters of the model. Each hyperparameter considered will be

Hyperparameter	Value
Optimizer	RMSprop
Learning-rate	0.003
Activation function	ReLU
Training epochs	100
Sliding window size	5

Table 5.1.: Initial hyperparameter configuration.

statistically evaluated. The experiments build upon one another. The results of an evaluation will be used in the following experiments. For the order of hyperparameter derivation exists no golden road. In literature different possibilities based on the authors favors could be found. The following order is based upon experiences gained in the POC phase and best practices seen during literature review.

In the beginning all hyperparameters are guessed based on the empirical knowledge from the POC. Adaptation takes place after each experiment. The initial hyperparameter configuration is assumed according to Table 5.1. This configuration was found producing promising results, so it is used as a starting point for the evaluation of the approach.

5.3.1. Network Architecture

During the POC several different network architectures were applied to the spare part data set. Some performed promising and some were not competitive compared to the current model. Busseti et al. [12] found in their research that the architecture has big impact on the performance of a deep learning based model. Therefore, the first evaluated hyperparameter is the topology of the proposed model.

Networks with a densely connected layer as first hidden layer showed good results during the POC, whereas networks with a recurrent layer as first one showed promising outcomes only in some cases. This could be explained by the capability of this layer type to transform the input data by projecting to another space and learning what values of the input vector deserve higher or lower weights. In case of the multivariate time series model one can argue that this first layer learns the auto-correlation between the demand and the VPD.

This leads to a densely connected layer as first hidden layer for all architectures. The following building blocks are used for the subsequent hidden layers:

- **RD:** Elman layer followed by a densely connected layer
- **LD:** Long Short Term Memory layer followed by a densely connected layer

These three different layer-types are placed between an input-layer containing a neuron for each element of the input vector and an output-layer with only one unit, returning the one-step-ahead forecast. To downsize the space of possible network topology constraints according width and depth are set up. The depth is limited to either 3, 5 or 7 hidden layers. The width of each layer is limited to 5 different values. Based on these constraints all permutations of above described layer types with a first hidden layer fixed as densely connected, five different widths and three depths could be calculated according to Equation 5.7, where $d = (3, 5, 7)$ are the different depths and $w = 5$ is the number of different widths.

$$a = \sum_d w^d 2^{\frac{d-1}{2}} \quad (5.7)$$

This results in a total of 637,750 possible network architectures. This is by far too much to statistically test all topology with the available computational resources. To overcome this infeasibility 1000 random sampled architectures will be run three times on a sample of 40 parts. The 50 best models then will be evaluated statistically and run 31 times on another 40 parts sample. This solution was favored over the strategy to thoroughly test 150 architectures, because it was preferred to touch a wider range of the space of topology, compared to the in-depth tested smaller range, even though results of three runs could be achieved by coincidence, which shall be overruled by the subsequent detailed test. Both approaches approximately use the for this experiment at maximum possible computational calculation time.

A depth of three hidden layers is regarded as on the border to deep learning, but as in favor of a less complex model this depth is taken into account. Deeper networks than five and seven hidden layers were neglected, because it is assumed that not enough data is available for training of such a model. Furthermore, the space of topology grows exponentially related to the network depth. The latter one also holds for more possible values related to the network width. Less potential width values are dismissed because they will restrict the

Depth	Layer	DL-STPM-VPD	DL-STPM
3	H1	15, 18, 22, 26, 30	5, 7, 10, 13, 15
	H2	9, 11, 13, 15, 18	3, 5, 7, 9, 11
	H3	3, 5, 7, 9, 11	3, 5, 7, 9, 11
5	H1	15, 18, 22, 26, 30	5, 7, 10, 13, 15
	H2	11, 13, 15, 17, 19	5, 7, 9, 11, 13
	H3	8, 10, 12, 14, 16	4, 6, 8, 10, 12
	H4	5, 7, 9, 11, 13	3, 5, 7, 9, 11
	H5	3, 5, 7, 9, 11	2, 4, 6, 8, 10
7	H1	15, 18, 22, 26, 30	5, 7, 10, 13, 15
	H2	13, 15, 17, 19, 21	5, 7, 9, 11, 13
	H3	11, 13, 15, 17, 19	4, 6, 8, 10, 12
	H4	9, 11, 13, 15, 17	3, 5, 7, 9, 11
	H5	7, 9, 11, 13, 15	2, 4, 6, 8, 10
	H6	5, 7, 9, 11, 13	2, 3, 5, 7, 9
	H7	3, 5, 7, 9, 11	2, 3, 4, 5, 6

Table 5.2.: Possible network widths per layer for each depth.

model more than the chosen setup. Table 5.2 shows all possible widths for each layer per model depths. The width of the first hidden layer is derived according to the dimension of the input vector. In case of the DL-STPM-VPD model the input contains three different time series, each for five time steps because of the sliding window size, resulting in a minimal width for the first hidden layer of 15. The maximal width 30 is twice the dimension of the input vector. In case of the DL-STPM the minimum width of the first hidden layer is equal to the dimension of the input vector and the maximum width is three times the input-dimension. The widths for the first hidden layer in-between the minimum and maximum are equally distributed. The first hidden layer is the same for all depths. All other layers are equally distributed according to a funnel-like shape, benefiting architectures with a wider first hidden layer among other things, narrowing with each layer until the last one. This should support the change of dimension from the multi-dimensional input to a one-dimensional output, which is the predicted next time-step.

Using the layer type building blocks and the widths, for each depth all permutations for both models are created. From these permutations respectively 40 3-layer, 480 5-layer and 480 7-layer architectures are randomly sampled per

	Architecture	Score		Architecture	Score
1	DRDRD-30-17-16-7-9	82376	26	DRDLD-26-19-10-7-9	75443
2	DRDLD-22-13-16-9-11	81833	27	DLDRDLD-26-19-19-13-15-11-9	75412
3	DRDRD-22-17-12-5-11	80815	28	DRDLDLD-30-19-19-11-15-11-7	75408
4	DRDRD-26-13-16-13-7	80530	29	DRD-18-13-11	75381
5	DRDRD-18-17-12-11-11	80262	30	DRDRD-30-19-14-5-5	75380
6	DRDRD-18-15-8-7-11	79361	31	DLDRDLD-30-17-13-9-13-13-11	75374
7	DRDRD-15-13-12-9-11	78670	32	DRDRD-15-19-12-11-11	75300
8	DRD-26-9-9	78542	33	DLDRD-15-11-8-9-7	74875
9	DRDRD-15-11-16-9-9	78438	34	DRDLDLD-15-21-19-9-15-13-3	74794
10	DRDRD-26-17-8-7-9	78412	35	DRDRD-30-19-14-5-9	74742
11	DRDRD-15-17-10-11-7	78391	36	DRDRD-30-11-10-7-11	74737
12	DRD-26-11-11	77815	37	DRDRD-18-13-10-11-9	74511
13	DRDLDLD-30-21-19-17-13-9-5	77748	38	DRDLD-26-11-12-9-5	74469
14	DRDLD-15-11-8-11-7	77725	39	DRDRD-15-11-14-11-5	74370
15	DRDRD-22-17-14-7-9	77545	40	DRDRD-26-15-12-11-11	74279
16	DRD-18-15-11	77403	41	DRDRD-26-19-12-7-7	74200
17	DRDRD-22-19-8-13-7	76936	42	DRDRD-22-19-14-9-7	74076
18	DRDRD-18-13-8-11-9	76934	43	DRDLDLD-26-21-19-15-11-11-7	74017
19	DRDRD-30-13-8-5-7	76756	44	DRD-26-18-9	73992
20	DRDRD-15-19-12-13-11	76622	45	DRDLDLD-30-21-19-11-9-11-5	73930
21	DRDRD-15-11-10-7-7	76347	46	DRDLD-26-13-12-11-7	73926
22	DRDRD-26-15-16-11-5	76099	47	DRDRD-15-17-14-13-7	73897
23	DRD-15-13-5	75881	48	DRD-30-11-5	73732
24	DRD-15-18-7	75619	49	DRDRD-15-11-12-13-7	73726
25	DRDRD-15-13-12-13-9	75576	50	DRDRD-22-11-8-7-11	73651

Table 5.3.: Ranking of 50 best architectures for DL-STPM-VPD.

model. The sample proportion is based on the size of each topology space and the favor of less complex solutions, by proportional covering a larger share for less complex architectures. These 1000 architectures are run three times each. According to the tournament scoring system described in section 5.2.1 a ranking of the tested topology is created. The ranking of the best 50 architectures is given in Table 5.3 and 5.4 respectively. Each architecture is described by the layer-types, D for densely connected, R for Elman layer and L for LSTM, followed by the widths of each layer, separated by $-$ symbol. The maximal reachable score in the tournament ranking was 120,000 (1000 models \times 3 evaluation functions \times 40 parts).

These 50 architectures for each model are run on another spare part sample to statistically evaluate, which network topology should be chosen for subse-

	Architecture	Score		Architecture	Score
1	DRDRD-7-7-12-5-4	82449	26	DRDLD-7-5-8-9-6	75418
2	DRDLD-5-7-10-7-10	81528	27	DRDRD-10-5-10-5-10	75384
3	DLDRDLD-15-9-6-3-10-9-2	78976	28	DRDLDDL-15-13-6-11-10-3-6	75378
4	DRDRDRD-15-9-12-11-6-5-6	78117	29	DLDDLDDL-15-13-10-9-10-3-6	75282
5	DRDLD-13-9-10-5-10	77637	30	DRDRDRD-15-13-10-3-4-7-6	75249
6	DRDRDLD-13-5-12-3-2-5-5	77358	31	DRDLDRD-7-9-4-5-4-2-3	75186
7	DRDLD-5-7-4-7-8	77236	32	DRD-5-9-11	75139
8	DRDLD-13-9-12-3-8	77135	33	DLDLDRD-13-9-12-11-4-2-4	75131
9	DRDLD-10-7-4-7-10	77109	34	DRDLD-5-5-8-3-6	75049
10	DRDLDRD-10-9-4-11-10-9-5	76971	35	DRDRD-7-11-6-5-10	74939
11	DRDLD-15-9-10-3-10	76736	36	DRDRD-7-13-12-3-10	74913
12	DRDLD-10-13-4-7-10	76725	37	DLDDLDDL-15-5-6-11-4-2-6	74788
13	DRDLDDL-15-11-4-11-8-7-5	76513	38	DRDRDRD-13-7-8-3-4-7-6	74720
14	DRDLDRD-10-11-12-7-10-3-3	76415	39	DLDDL-10-7-6-9-2	74653
15	DRDLD-7-5-4-7-8	76409	40	DRDLDDL-5-9-12-7-6-3-6	74639
16	DLDDL-13-11-8-3-2	76372	41	DRDRDLD-7-5-8-7-2-5-3	74571
17	DRDRDLD-10-5-8-3-8-3-4	76171	42	DLD-7-5-9	74528
18	DLDRDLD-5-13-10-11-10-2-5	75945	43	DRDLD-15-13-4-9-10	74521
19	DRDRD-13-11-4-5-2	75869	44	DRDLD-15-7-10-7-6	74428
20	DRDLDDL-10-11-12-5-6-2-2	75662	45	DRDRD-10-9-6-3-2	74372
21	DLDDLDDL-15-9-12-7-10-2-3	75650	46	DLDRD-15-11-6-7-4	74301
22	DRDRD-10-13-4-5-4	75537	47	DRDLDRD-13-7-10-5-6-7-3	74298
23	DRDLD-10-11-10-9-4	75528	48	DRDRDRD-7-9-12-11-2-9-3	74280
24	DLDRD-5-9-6-5-4	75495	49	DLDRDRD-13-9-10-5-2-5-2	74263
25	DRDRDRD-13-11-8-5-10-2-4	75450	50	DLDDLDDL-5-11-10-11-2-5-5	74233

Table 5.4.: Ranking of 50 best architectures for DL-STPM.

quent experiments. From the results the median performing network run of the 31 runs, according to RMSE, of each tested architecture is selected for tournament ranking. By tournament ranking the best performing architecture for each part, according to RMSE, Chi-Squared-Distance and CC is found. This best performing architecture on each part is used as reference to calculate the significance of the results, as described in section 5.2.3. Therefore, the error vector of the best topology of a part is compared by significance test to the error vectors of the other architectures, resulting in a $m \times n$ matrix of p-values, where m is the number of parts of the sample and n is the number of architectures tested. The matrix is used to calculate ψ_{bm} and ψ_{sp} for this particular experiment. The results of the significance evaluation can be found in Appendix Table A.1 and A.2 respectively.

	Architecture	ψ_{bm}	ψ_{sp}
1	DRD-26-18-9	90	12
2	DRD-30-11-5	88	21
3	DRD-18-15-11	73	8
4	DRD-26-11-11	58	13
5	DRDRD-30-17-16-7-9	49	14
6	DLDRD-15-11-8-9-7	48	15
7	DRDRD-15-19-12-11-11	48	16
8	DLDRDL-26-19-19-13-15-11-9	42	25
9	DRDRD-18-17-12-11-11	35	16
10	DRDL-22-13-16-9-11	32	13
11	DRDL-26-21-19-15-11-11-7	30	22
12	DLDRDL-30-17-13-9-13-13-11	27	21
13	DRDRD-15-11-10-7-7	23	17
14	DRDRD-30-19-14-5-5	22	20
15	DRD-15-18-7	19	16
16	DRDRD-22-11-8-7-11	17	15
17	DRD-18-13-11	15	10
18	DRDRD-30-11-10-7-11	14	16
19	DRDRD-15-19-12-13-11	13	15
20	DRDL-26-13-12-11-7	12	18
21	DRDRD-22-19-8-13-7	12	16
22	DRDRD-22-17-12-5-11	10	16
23	DRDRD-22-17-14-7-9	9	14
24	DRDRD-15-17-14-13-7	8	16
25	DRD-15-13-5	5	19
26	DRD-26-9-9	5	12
27	DRDRD-30-19-14-5-9	1	11

Table 5.5.: Significance ranking of 27 best architectures for DL-STPM-VPD.

Based on the significance test a ranking of the best models can be created. Table 5.5 shows the ranking according to ψ_{bm} for the DL-STPM-VPD model. Due to convenience only architectures that achieved a ψ_{bm} score greater zero are displayed. The maximal reachable ψ_{bm} score for this experiment is 1960. A 3-layer ANN, composed of a densely connected, an Elman and a densely connected layer performed best on the given sample of spare parts. It makes use of a funnel shape width, representing the transformation from multi-dimensional input to one-dimensional output. Most of the top ten architectures show this structure, underpinning the benefit of this hypothesis. Furthermore, the top four architectures are all 3-layer topology. This indicates that a simpler ANN is capable of learning the multivariate time series features, whereas more complex structures tend to have problems, for example the 7-layer architectures are usually significantly worse than the best architecture of a particular part for half of all spare parts of the evaluated sample. It also seems that Elman layer is preferred over LSTM layer. One could argue that the low amount of training data may be a reason for this. Probably the LSTM was not capable of finding the time dependent relations within that few training data. Furthermore, the best model, DRD-26-18-9 has a ψ_{sp} value of 12, meaning it performed significantly worse than the best architecture only on 12 out of 40 parts of the sample. This supports the selection of this topology because this is the 4th-lowest value.

The ψ_{bm} ranking resulting from significance test for the model without VPD is shown in Table 5.6. Due to convenience only models that achieved a ψ_{bm} score greater zero are listed. For this time series problem also a 3-layer architecture, with same layer-types as for the DL-STPM-VPD model, but with different widths performed best. The structure formed by the width of the layers is inverted compared to the model for the multivariate time series problem. It goes from 5 over 9 to a width of 11 for the last hidden layer. There is no funnel like structure recognizable at the best architectures, concluding the hypothesis of transforming from multi-dimensional input space to a lower-dimensional space does not hold for the univariate time series problem. One reason for this may be the lower dimension of the input vector, compared to the multivariate time series. The model seems to learn a representation of the time series in a higher-dimensional space than the input space. In general, the mixture of architectures, which performed promising is more diverse, compared to the DL-STPM-VPD model. One could argue that this underpins the difficulty of the task to learn a model based only on few information. In contrast to

	Architecture	ψ_{bm}	ψ_{sp}
1	DRD-5-9-11	111	10
2	DRDLD-7-5-4-7-8	80	15
3	DRDLD-10-13-4-7-10	71	12
4	DRDLD-7-5-8-9-6	50	14
5	DLDLDL-15-5-6-11-4-2-6	45	17
6	DRDRD-7-13-12-3-10	38	11
7	DRDLD-13-9-10-5-10	37	13
8	DLDRD-15-11-6-7-4	34	20
9	DRDRDL-7-5-8-7-2-5-3	34	16
10	DRDRDRD-13-11-8-5-10-2-4	34	16
11	DRDLD-10-7-4-7-10	32	13
12	DRDLDL-10-11-12-5-6-2-2	31	17
13	DRDLD-5-7-10-7-10	27	14
14	DRDRD-7-11-6-5-10	27	10
15	DLDLDL-15-13-10-9-10-3-6	26	20
16	DRDLD-15-9-10-3-10	25	10
17	DLDL-13-11-8-3-2	23	19
18	DLDLDL-15-9-12-7-10-2-3	20	16
19	DLDRDL-15-9-6-3-10-9-2	17	19
20	DRDRD-10-13-4-5-4	16	12
21	DRDLDL-15-11-4-11-8-7-5	11	19
22	DRDRDL-10-5-8-3-8-3-4	8	15
23	DRDRD-7-7-12-5-4	6	13
24	DRDLDL-5-9-12-7-6-3-6	3	14

Table 5.6.: Significance ranking of 24 best architectures for DL-STPM.

the multivariate model, where several 3-hidden-layer architectures with the same layer-types were under the most successful, there is no architecture type superior to the other for the model without VPD. Nonetheless, the DRD-5-9-11 architecture has with a ψ_{sp} value of 10 one of the lowest number of models it is significantly worse compared to. This underlines that this rather simple topology has better generalization capabilities than the other best performing architectures, which are significant worse for more than 10 models, supporting the decision to continue evaluating this topology.

5.3.2. Optimizer and Learning-rate

Based on the results from the previous section the optimizer of the network weights and the related learning-rate are experimentally derived in the following. As of Bengio [5], the learning-rate and the optimization algorithm are two very important hyperparameters of model training. The in Section 3.3.1 introduced optimization algorithms, Stochastic Gradient Descent, Adam and RMSprop are evaluated with different learning-rates, that are derived from the default learning-rate of Adam and RMSprop: 0.001. During the POC a higher learning-rate than the default was applied. Therefore, a lower rate is regarded as not promising, why 0.0005 as half of the default rate is applied as lower bound. Furthermore, 10 times the default learning-rate and two values in-between, 0.0033 and 0.0066 are evaluated. Finally 0.1, the default rate by factor 100 as upper bound and 0.05 as the mean of both latter multiples of the default learning-rate is used. The proposed learning-rates logarithmic amplify, originating from the default rate. The distance between two successive learning-rates increases with the basis of the logarithm. This strategy favors rates in similar range as the default learning-rate, but also includes larger rates, even though the risk of gradient oscillation increases. Smaller learning-rates, than the chosen ones are neglected. Based on the experiences from the POC, lower rates than the default learning-rate do not reach the (local) optimum within the available training epochs. This behavior is expected to be statistically proven, therefore only one rate smaller than default will be tested. Due to limitation of computational resources a more detailed test, e.g. the relation between learning-rate and training-epochs, could not be conducted.

Each optimizer is evaluated with each of the seven learning-rates. Therefore, the models of the previous section are run 31 times, each on a third 40

spare parts sample. Any of the 21 above described optimization algorithm / learning-rate combinations are tested for DL-STPM-VPD model, as well as for the DL-STPM model. Besides the learning-rate default parameter for other optimizer parameters, as described in section 3.3.1, are applied to the optimizer. As in the network architecture section the best performing configuration for each part is determined by tournament ranking evaluation. Furthermore, the significance ranking is calculated the same way, based on the RMSE error vectors as described in the previous section, to get the ψ_{bm} and ψ_{sp} values. For the experiments to determine the optimizer and learning-rate combination a maximal ψ_{bm} score of 800 is possible. The results of the significance test can be found in Appendix Table A.3 and A.4.

The ranking of optimizer and learning-rate combinations for the DL-STPM-VPD model can be found in Table 5.7. The Adam algorithm with a learning-rate of 0.01 performed superior to the other configurations. Adam outperforming SGD confirms literature review because of the adaptive learning-rate as enhancement. RMSprop performed competitive. If regarded on a learning-rate level RMSprop is clearly better performing than SGD for the same learning-rate, but Adam is less often significantly worse than RMSprop on the same learning-rate. This indicates that the estimation of the decay rate performed by Adam is more suitable to the data than that of RMSprop. Furthermore, it may also be possible that Adam can better handle the low amount of training data than RMSprop, which should be scientifically examined in another study.

The learning-rate of 0.01 may also depend on the amount of training data. A probable explanation of a rate, ten times the default learning-rate is that because of a smaller number of data points, larger steps along the gradient are preferred over smaller ones. For Adam and RMSprop the ψ_{sp} value grows if the learning-rate falls under 0.0066 or if the rate goes beyond 0.01, e.g. Adam with learning-rate of 0.01 is significant worse than seven other configurations. If the learning rate becomes 0.05 this number raises to 38 and in case of 0.0033 it grows to 19. This indicates that the range between both latter mentioned learning-rates lies within a (local) minimum that should be further evaluated in more detail. The previously mentioned effect applies for Adam and RMSprop. Due to the limitations for this study, this needs to be postponed to a later research. This also holds for the relation of learning rate and training epochs. It remains an open issue whether a smaller learning-rate would perform better, if it has more time for training than in the current setup.

	Learning-rate	Optimizer	ψ_{bm}	ψ_{sp}
1	0.01	Adam	229	7
2	0.01	RMSprop	84	16
3	0.0066	RMSprop	73	15
4	0.0033	RMSprop	53	14
5	0.0066	Adam	43	6
6	0.001	Adam	31	25
7	0.0033	Adam	29	19
8	0.01	SGD	16	35
9	0.05	RMSprop	13	39
10	0.0066	SGD	11	34
11	0.0005	SGD	0	39
12	0.001	SGD	0	39
13	0.0033	SGD	0	36
14	0.05	SGD	0	31
15	0.1	SGD	0	28
16	0.0005	Adam	0	30
17	0.05	Adam	0	38
18	0.1	Adam	0	38
19	0.0005	RMSprop	0	28
20	0.001	RMSprop	0	28
21	0.1	RMSprop	0	37

Table 5.7.: Significance ranking of optimizer / learning-rate for DL-STPM-VPD.

Table 5.8 summarizes the results of the significance test for the model without VPD. Adam outperformed the other optimization approaches for the univariate time series too, but with a slightly smaller learning-rate than for the DL-STPM-VPD model. RMSprop performed worst, indicating that its learning-rate adaptation needs more data than available. This is underpinned by SGD outperforming RMSprop without any learning-rate adaptation. If regarded the ψ_{sp} values Adam slightly outperformed SGD, outperforming RMSprop. The slightly smaller learning-rate of 0.0066 compared to the model with VPD may be explained by the lower dimension of the input space, resulting in smaller network width and less connections a weight needs to be learned for. Therefore, a smaller step-size along the gradient is possible in

	Learning-rate	Optimizer	ψ_{bm}	ψ_{sp}
1	0.0066	Adam	80	20
2	0.05	SGD	70	15
3	0.01	Adam	61	20
4	0.0066	SGD	47	22
5	0.001	SGD	36	23
6	0.01	SGD	35	20
7	0.001	RMSprop	31	15
8	0.0005	RMSprop	27	14
9	0.0005	SGD	17	27
10	0.0005	Adam	17	11
11	0.0033	SGD	14	21
12	0.0033	RMSprop	10	21
13	0.1	SGD	8	23
14	0.001	Adam	5	20
15	0.0033	Adam	0	19
16	0.05	Adam	0	21
17	0.1	Adam	0	25
18	0.0066	RMSprop	0	27
19	0.01	RMSprop	0	36
20	0.05	RMSprop	0	28
21	0.1	RMSprop	0	30

Table 5.8.: Significance ranking of optimizer / learning-rate for DL-STPM.

this case. Furthermore, an adaptation of the learning-rate still is preferable by outperforming the optimizer with a fixed learning-rate.

Even though Adam with a learning-rate of 0.0066 has achieved the highest ψ_{bm} value, itself performed significant worse on 50% of the parts, compared to the particular best models. This indicates that this configuration had problems learning time series features for some spare parts, but performed strong on others. Nonetheless Adam with a learning-rate of 0.0066 is preferred over the second-best configuration, which seems to have slightly better performance regarding the ψ_{sp} results, but outperformed less configurations, which is regarded as more important performance indicator in this case. The chosen model performed better than the mean, which was outperformed on round

about 22 out of 40 spare parts. This also indicates that the results are not that precise as for the model with VPD, originating in the difficulty of the task of univariate time series prediction. Further on, it may be noted that Adam and RMSprop with a learning-rate of 0.0005 performed competitive related to the best configurations per part. This presumed generalization capability of smaller learning rates in relation with the training epochs could be evaluated in detail in a future study.

5.3.3. Activation Functions

This section evaluates different combinations of activation functions on a fourth sample of 40 spare parts. As of Palit and Popovic [73] the activation functions are a connecting component of network architecture and the process of network training, influencing the network output and the backpropagated error. In section 3.3.1 four activation functions were introduced: Sigmoid, ReLU, leaky ReLU and SoftPlus. The latter three will be evaluated in the following. According to literature these are the most promising activation functions for deep learning and recurrent models. Due to limitation of computational resources not every combination of activation functions, distributed over the three hidden layers of both models could be examined. To overcome this limitation the activation functions are only permuted layer-type-wise. The same layer-type is combined with the same activation function over the whole model. For a model with three hidden layers, where two layers are of the same type, in our case densely connected, this results in nine different combinations of activation functions. For evaluation each combination is run 31 times, making use of the hyperparameters derived in the previous sections, to get the related error vectors for significance test. The results of the significance evaluation can be found in Appendix Table A.5 for the model with VPD input and Table A.6 for the model without VPD. A maximal ψ_{bm} value of 320 is reachable.

The summarized results from the significance test for the DL-STPM-VPD model can be found in Table 5.9, showing the applied activation function for each hidden layer and the related ψ_{bm} and ψ_{sp} values. A combination of leaky ReLU, as activation function for the densely connected layers and basic ReLU for the Elman layer outperformed the other combinations in terms of ψ_{bm} . If regarded for how many parts each model performed significantly worse than the best model for this part all configurations did not perform

	H1	H2	H3	ψ_{bm}	ψ_{sp}
1	leakyReLU	ReLU	leakyReLU	51	21
2	leakyReLU	SoftPLus	leakyReLU	32	25
3	ReLU	leakyReLU	ReLU	27	24
4	leakyReLU	leakyReLU	leakyReLU	26	18
5	SoftPLus	SoftPLus	SoftPLus	25	20
6	ReLU	ReLU	ReLU	21	18
7	SoftPLus	leakyReLU	SoftPLus	17	28
8	SoftPLus	ReLU	SoftPLus	5	21
9	ReLU	SoftPLus	ReLU	0	29

Table 5.9.: Significance ranking of activation functions for DL-STPM-VPD.

that well. The lowest ψ_{sp} value was achieved by ReLU or leaky ReLU applied for all three layers, with being significantly worse on 18 parts out of 40. The above-mentioned combination of leaky ReLU and ReLU was significantly worse than the best model on 21 spare parts, which is still one of the best values. This indicates that there is no combination of activation functions that has good generalization capabilities, clearly outperforming the others. Therefore, the combination, achieving the best ψ_{bm} result was chosen as hyperparameter configuration for further experiments. Even though the results do not strongly emphasize a combination of activation functions, leaky ReLU as activation function for densely connected layers seems to be a good choice, as most of the best performing configurations apply this activation function for this layer-type. A reason for that may be the small gradient, added by leaky ReLU even if the unit is not active. This may benefit the training in case of only few training data, resulting in better performance.

Table 5.10 summarizes the results of the significance evaluation for the model without VPD. A combination of leaky ReLU as activation function for all three layers significantly outperformed the most combinations of activation functions on parts it was found to be the best. This configuration also showed the best generalization capabilities and was only for 7 spare parts out of 40 found to perform significantly worse than the best model per part. This is one of the best ψ_{sp} values among all combinations of activation functions, underpinning the superiority of this model. In case of the univariate time series the advantage of also having a small gradient if the unit is not active seems to benefit the training even more than in case of the multivariate time series problem, as

	H1	H2	H3	ψ_{bm}	ψ_{sp}
1	leakyReLU	leakyReLU	leakyReLU	40	7
2	SoftPlus	leakyReLU	SoftPlus	32	10
3	ReLU	ReLU	ReLU	14	19
4	SoftPlus	SoftPlus	SoftPlus	14	19
5	ReLU	leakyReLU	ReLU	12	7
6	ReLU	SoftPlus	ReLU	7	16
7	SoftPlus	ReLU	SoftPlus	7	18
8	leakyReLU	SoftPlus	leakyReLU	4	18
9	leakyReLU	ReLU	leakyReLU	3	19

Table 5.10.: Significance ranking of activation functions for DL-STPM.

the leaky ReLU activation function is used for all hidden layer. This may be explained by the even less information available for training, compared to the time series containing VPD. Furthermore, leaky ReLU is used for the recurrent layer for all of the models, showing the best generalization capabilities. In opposite to the model with VPD some combinations of activation functions are superior to the other configurations, achieving better ψ_{bm} and ψ_{sp} values than other configurations. This states that in case of the univariate time series some combinations of activation functions, like leaky ReLU for all layers, can handle the data better than other by showing superiority in terms of ψ_{bm} and satisfactory generalization by a good ψ_{sp} value. This may originate in the one-dimensional nature of the time series, implying less relations within the data, making it easier to find a combination capable of dealing with the underlying structure of the data producing processes.

5.3.4. Sliding Window Size

The size of the sliding window that is moved through the input data is covered in the following. This hyperparameter controls the format of the input data. As of Goodfellow et al. [40], model tuning related to the input data has also great impact on the performance of a deep ANN. The size of the sliding window defines how many x_t are combined in the input vector of a single time step. It specifies the dimension of the input vector, e.g. in case of a window size $w = 3$ the input for each time step is composed out of x_{t-2} , x_{t-1} and x_{t-0} . This window is then moved through the data like a queue, adding the

	window size	ψ_{bm}	ψ_{sp}
1	w=3	24	9
2	w=2	24	12
3	w=4	18	10
4	w=8	12	16
5	w=9	11	16
6	w=5	10	13
7	w=7	8	16
8	w=6	0	15

Table 5.11.: Significance ranking of sliding window sizes for DL-STPM-VPD.

youngest value on the one end and removing the oldest value on the other end. The larger the sliding window, the more information is aggregated in one input vector, but the less data is available for network training because the number of available data points for training is always reduced by the size of the sliding window. The sliding window approach adds time related information to each input data point by transforming it to a higher space, additionally containing information of past time steps. In time series literature this approach is usually applied for simplification of learning of time dependent relations by artificially providing more information for each time step.

The impact of the sliding window size on both models will be evaluated in the following. Therefore each model is run on a fifth sample of 40 spare parts with different sliding window sizes from the interval $[2, 9]$. Two is the smallest possible window size and nine is regarded as maximum, minimizing the amount of training data drastically. To calculate statistical significance each configuration is again run 31 times to get the appropriate error vector. Because of the number of tested window sizes a maximal ψ_{bm} score of 280 could be achieved in the significance evaluation. The results of the significance test of the sliding window size experiment can be found in Appendix Table A.7 and A.8 for the model without VPD respectively.

Table 5.11 summarizes the results of the significance test of the DL-STPM-VPD model. The sliding window sizes two and three performed equally, regarded how many models were found to be significantly worse for spare parts these models performed best. This indicates that a smaller sliding window performs better because of more available data points for network training.

	window size	ψ_{bm}	ψ_{sp}
1	w=2	65	13
2	w=9	21	28
3	w=3	19	15
4	w=6	15	16
5	w=4	10	10
6	w=5	6	15
7	w=7	6	23
8	w=8	6	28

Table 5.12.: Significance ranking of sliding window sizes for DL-STPM.

This theory is supported by the ψ_{sp} results. Originating from a sliding window size of three, the number of models that performed superior steadily increases with the window size increasing. Also, a sliding window size of two is outperformed on 12 parts, which are more parts than in case of a window size of three. Therefore, the latter configuration is regarded as best model of this experiment. Nonetheless, the results of this evaluation questions the sliding window approach at all. If the highest amount of training data is preferred, the model should be evaluated without any sliding window applied to the input data in future research. Furthermore, it may be questioned if the input data should be extended by the time dependent relations or whether the model should not be constrained beforehand, instead learning the relations by its own from more available data.

The results of the sliding window experiment for the model without VPD are shown in Table 5.12. In case of the univariate time series the sliding window approach is also questioned by the outcome of the evaluation at all. A window size of two clearly outperformed the other configurations in terms of ψ_{bm} . Also, if regarded on the performance of the models over all parts, all, except a sliding window of size four were significant worse than the respectively best model on more than 13 spare parts, which is the result for the sliding window of size two. Furthermore, the performance decreases related to the window size increase is verified for the univariate time series problem in more clarity. This concludes that the restriction by artificially increased time related input does not benefit the model. One can argue that the model should learn this relation by itself, without any constraints made by the input. These findings could be further evaluated and proven in a future study.

5.3.5. Data Augmentation

The following section covers the evaluation of the data augmentation step. According to Goodfellow et al. [40] this step is highly recommended during optimization of a deep learning model. Data augmentation adds artificial data to the training data. On the one hand side this should evaluate the influence of more data for model training and on the other hand side the generalization capabilities of a model shall be strengthened. The latter effect would require the addition of noise data to the time series or to extend the multivariate time series by additional features. Due to the few training information available, complexity reasons and the difficulty to extend the time series in a beneficial way the latter data augmentation approach is abandoned. The first mentioned impact is discussed in the following.

To artificially extend the data the mean of two successive time steps x_t and x_{t+1} is added in-between them, as defined in Equation 5.8. This process can be recursively repeated, extending the time series to length T_a , according to Equation 5.9. T is the original length of the time series and d is the depth of recursion, starting at one with the first artificial extension. It may be noted that this kind of augmentation can be regarded as data smoothing. The data is smoothed by stretching along the time dimension. Nevertheless, it adds data points to the training data and the influence could be evaluated. Artificially extension of the time series beyond the proposed method becomes a tough task that could be reliably solved only if more information about the data generating processes is available.

$$x_a = \frac{x_t + x_{t+1}}{2} \quad (5.8)$$

$$T_a = (2^d T - 2^{d-1}) - 1 \quad (5.9)$$

For evaluation each model is run 31 times on a sixth sample of 40 spare parts. The models are configured according to the above derived hyperparameters. No augmentation, represented by a degree of zero, data augmentation of degree one and two are compared. Greater recursion depths are neglected because of the smoothing character of the applied augmentation, assuming no further information gain. The models are trained on the augmented data. For prediction the original input is used, otherwise the prediction horizon would be reduced by the degree of augmentation, resulting in a less accurate forecast for the same period because more time steps need to be predicted, if the horizon is

	Degree	ψ_{bp}	ψ_{sp}
1	d=0	24	6
2	d=2	6	17
3	d=1	5	12

Table 5.13.: Significance ranking of data augmentation for DL-STPM-VPD.

extended. The results of the significance evaluation could be found in Table A.9 and A.10 respectively, resulting in a maximal ψ_{bp} value of 80.

Table 5.13 shows the results of the evaluation of the data augmentation process for the model with VPD. The results clearly suggest that data augmentation does not benefit the model. A degree of zero outperformed the other tested configurations in terms of models that were found to perform significantly worse on parts the model without data augmentation performed best. This also holds if compared, for how many parts a model performed significantly worse than the best model for that particular part. The number of models by whom a particular configuration is outperformed steadily grows with the degree of recursion of the data augmentation.

Reasons for this result could be manifold. The current hyperparameter configuration was derived on not smoothed data. The influence of the augmentation on the data's structure the model is capable of learning could be that strong, that it results in a performance decrease if this structure changes to a certain degree. Furthermore, the model could already be overfitting the data structure and therefore lack in generalization. Last but not least, the augmentation could add the wrong information to the data. The timely stretched data could influence the time related pattern, like trend or seasonality in a way, that the model cannot learn to transform these relations to the original input data. The first and last-mentioned explanation, which are related by certain degree, seem most plausible. Because of the low amount of information that is available for training the model is sensible to changes of these. Overfitting, as justification of the results is discarded for the moment, because countermeasures, like the exchange of the samples for each particular evaluation should protect against it. Nonetheless, overfitting should not be dismissed totally and the influence of changes to the input data should be evaluated in a future study.

Table 5.14 summarizes the results of the significance test for the data augmentation evaluation for the DL-STPM model. The outcome is similar to

	Degree	ψ_{bm}	ψ_{sp}
1	d=0	15	8
2	d=2	9	7
3	d=1	1	10

Table 5.14.: Significance ranking of data augmentation for DL-STPM.

the model with VPD but not that severe. Even though a degree of zero performed best in terms of models that performed significantly worse on spare parts no data augmentation achieved the best results, augmentation of degree two is competitive, according to its ψ_{sp} score. No augmentation is preferred because it was found to be the best model according to the tournament ranking based on the evaluation functions on 50% of the parts contained in the sample, whereas a degree of two only was found to be the best one on 13 parts. Thus, the question, if the DL-STPM model could be supported by data augmentation remains unacknowledged and should be investigated in more detail in a later study. Based on the results it could not be stated if artificial changes to the training data benefit the model or not, which underpins the toughness of the univariate time series problem in general.

5.3.6. Training Epochs

The last hyperparameter evaluated is the number of training epochs. It controls for how many iterations the training data is completely processed through the ANN, to learn the connection weights. In an ideal case the optimization algorithm finds the global optimum within the given training epochs. Often this process gets stuck in local optima and the error of the ANN is not minimized further, because the training algorithm cannot get out of the local optimum. Further training iterations after the optimization reached the local optimum do not change the network output significantly and training could be aborted. This strategy, called early stopping, could be automated e.g. by a network error threshold or a number of iterations, the error did not change noticeable. If such a threshold is reached, training is stopped early. Nonetheless, it is difficult to derive a threshold value. In case of this study, where an ANN for each spare part is trained, it is more useful to derive the at least needed number of training epochs and accept possible useless iterations, than trying to find a

	Training epochs	ψ_{bm}	ψ_{sp}
1	e=70	30	8
2	e=200	11	11
3	e=100	6	8
4	e=400	4	12
5	e=800	4	16

Table 5.15.: Significance ranking of training epochs for DL-STPM-VPD.

general early stopping criterion, because it is regarded as more important to ensure quality of the results, than minimization of computational effort.

During the POC the prototype networks usually reached a local optimum within 70 to 90 epochs. Based on these experiences the number of training epochs e was set to 100 for the previous experiments. For the current evaluation 70, 100, 200, 400 and 800 training epochs are tested. 70 is regarded as minimum, based on the empirical knowledge from the POC. 100 as hyperparameter used for the previous evaluation steps is also considered. Furthermore, the number of epochs is doubled, until a maximum of 800 is reached. The two biggest values are expected to bring no improvements to the training process anymore.

As usual, each configuration will be run 31 times on a fresh sample of 40 spare parts for the DL-STPM-VPD and DL-STPM model, configured based on the derived hyperparameters. According to the different configurations and the size of the spare part sample a ψ_{bp} score of maximal 160 could be reached in the significance test. The results of could be found in Appendix Table A.11 for the multivariate time series and A.12 for the univariate demand data respectively.

Table 5.15 summarizes the results of the significance test for the multivariate time series model. 70 training epochs clearly outperform the other configurations in terms of ψ_{bm} . Concerning spare parts, a model performed significantly worse than the best configuration on this part, the two smallest numbers of training epochs performed equally well. For the other tested number of epochs the performance decreases proportional to increase of iterations. This confirms the expectation that a larger amount of training epochs will not benefit the model. Due to the few training data the model relatively fast reaches a local optima. Further training rather increases the training error by ending on the

	Training epochs	ψ_{bm}	ψ_{sp}
1	e=200	40	25
2	e=800	29	31
3	e=70	22	17
4	e=100	18	21
5	e=400	12	27

Table 5.16.: Significance ranking of training epochs for DL-STPM.

borders of the local optima, but not at the actual local minimum. In case a larger number of training epochs performed best on a particular part, often the other configurations did not perform significantly worse. This also underpins that a larger number of iterations does not bring any advantage, compared to the found best amount $e = 70$. Anyhow, an approach making use of early stopping should be evaluated in a subsequent study, to check, whether a dynamical approach could bring a larger benefit than a fixed number of epochs. This study could incorporate the findings of these experiments, to ensure the usually at least needed number of training epochs in case the early stopping criteria is not reached.

The results of significance evaluation for the model without VPD can be found in Table 5.16. For the univariate time series the outcome of the experiment differs from the above discussed. 200 training epochs significantly outperformed the most models on spare parts it were found to perform best. If regarded for how many spare parts a configuration performed significantly worse than the best model for each part, the tendency is the same. The higher the number of training epochs gets, the higher the ψ_{sp} value. It may be noted that the ψ_{sp} values are approximately twice as high as for the multivariate time series experiment. As well, the differences between configurations for each part are more severe than for the DL-STPM-VPD model. In case of the univariate time series the best configuration for a spare part more often significantly outperformed the other models on that particular part, than it was the case for the time series containing VPD. This indicates that the generalization based on less information is more difficult. Concluding, this supports the hypothesis that it is difficult to find a hyperparameter set with desirable generalization capabilities for the univariate time series problem, which may be lead back to the small amount of data available, to derive knowledge from.

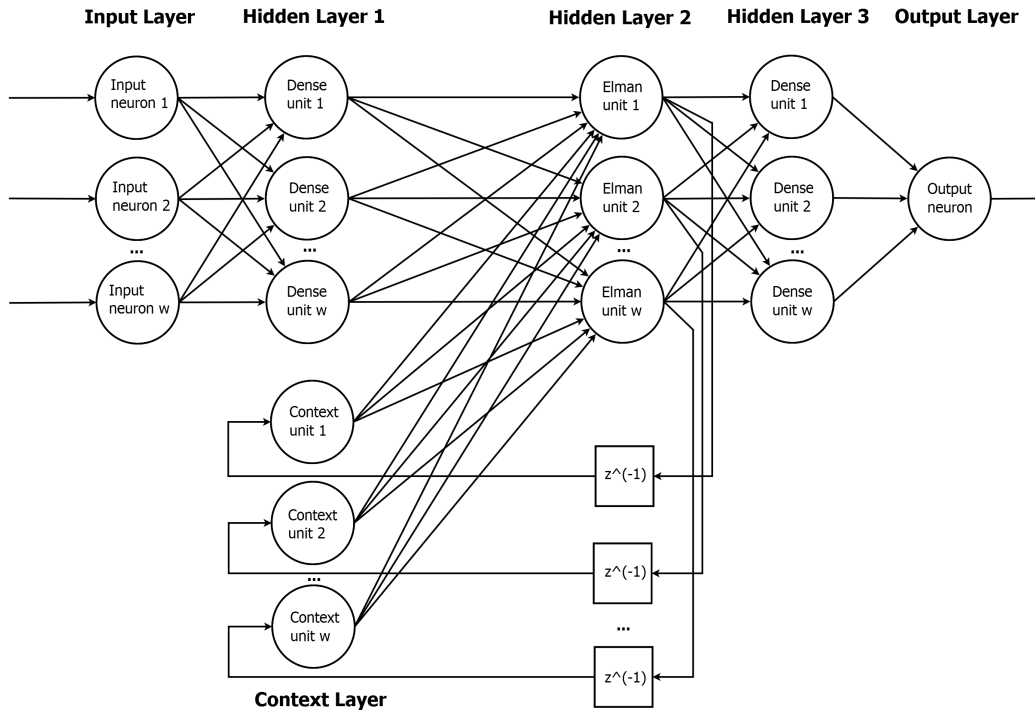


Figure 5.2.: Exemplary network structure.

5.4. Summary

The in the previous sections derived hyperparameters are only a selection that is regarded as containing the most important ones. There are several more that could be evaluated and tuned, like further hyperparameter for network training, e.g. momentum, or regularization strategies and so on. The experiments could be extended by a broader range of values or options. Nevertheless, the evaluated hyperparameters are regarded as a solid mixture of architecture and training optimization and the available resources, with respect to computational time for experiments, were fully used. A different order of the tests or different configurations may have led to other results. The optimal technique for hyperparameter estimation for deep learning is still an active research topic and a not yet solved problem.

Figure 5.2 exemplary shows the architecture of the deep ANN, which is regarded as the most important hyperparameter. Both models, with and without VPD make use of the same topology, as visualized by Figure 5.2, mere with different widths for each model. Table 5.17 summarizes the experimental

Hyperparameter	DL-STPM-VPD	DL-STPM
Architecture	Densely connected, Elman, Densely connected	Densely connected, Elman, Densely connected
Optimizer	Adam	Adam
Learning-rate	0.01	0.0066
Activation function	leaky ReLU (H1), ReLU (H2), leaky ReLU (H3)	leaky ReLU (H1), leaky ReLU (H2), leaky ReLU (H3)
Training epochs	70	200
Sliding window size	3	2
Data augmentation	no	no

Table 5.17.: Experimentally derived hyperparameter configuration.

derived hyperparameters for both models. These configurations will be used for the evaluation of the proposed model in comparison to the current model and its enhancements, to answer the research hypothesis of this work in the following chapter.

6. Evaluation and Comparison of Proposed Models

This section compares the currently by IBM applied model, it's in section 4.3 proposed enhancements and the in this study derived deep learning model using in the previous section derived hyperparameter configurations, to answer the research question of this thesis. Each model is evaluated on a set of 365 spare parts, sampled from the multi- and univariate time series data respectively. Because of the larger samples than the ones used for experimental hyperparameter estimation a more accurate prediction of the overall model performance could be done. According to Equation 5.5 for calculation of sample size the margin of error for a sample of 365 parts is 5%, with a confidence interval of 95%. This holds for both samples, even though the margin of error for the sample without VPD is slightly less than for the multivariate time series data because of the smaller number of parts, but this difference ranges in per mill region. The experiments will be repeated 31 times. A comparison is done by the same approach as for the experiments. First the best performing model according to the evaluation functions for each spare part is determined. Then the related p-values are calculated and the ψ_{bm} and ψ_{sp} values for each model are derived. According to the number of spare parts contained in the sample and the in the significance evaluation involved models a maximal ψ_{bm} score of 730 could be achieved.

6.1. DL-STPM-VPD

The ranking of the multivariate models can be found in table 6.1, aggregating the results from the significance test, which could be found in table A.13. The deep learning based approach performed superior to the enhanced STPM-VPD model, followed by the currently applied model in terms of ψ_{bm} . This

	Model	ψ_{bm}	ψ_{sp}
1	DL-STPM-VPD	296	180
2	STPM-VPD-enh	194	241
3	STPM-VPD	185	254

Table 6.1.: Significance ranking versus current model for DL-STPM-VPD.

confirms the research question, whether an ANN based approach is capable of predicting the spare part demand with higher accuracy for the model with Vehicle Production Data. This is also supported by the ψ_{sp} values. The deep ANN achieved with 180 the best result, followed by the enhanced STPM model. The worst ψ_{sp} value was measured for the currently applied STPM model. Concluding the results of the significance test, the deep learning based approach achieves a higher forecast accuracy in terms of RMSE as the currently applied model and its proposed enhanced version if regarded at the whole sample evaluated. Nevertheless, this could be stated only because the ANN was found to be the best model according to the evaluation functions on the majority of the parts. The high ψ_{sp} values indicate that the models were usually significant worse than the best model of a particular part. This suggests that in many cases, if the model was not found to be the best on a spare part according to the tournament ranking evaluation, it performed not competitive compared to the other. This means that the deep learning based approach could predict a larger number of parts than the currently applied model with higher accuracy, but performs not satisfying on all parts of the sample. Yet this is still an improvement to the currently applied model.

Figure 6.1 summarizes the direct tournament ranking comparison of either the currently applied model or the enhanced version of the STPM-VPD model against the deep learning based approach. For each part of the sample the results according to the three evaluation functions, RMSE, Chi-Squared-Distance and CC are compared. A model is considered as better than the other approach if it was found to outperform it for at least two out of three evaluation functions. In the end, it is counted for each model for how many parts of the sample it performed superior to the compared approach.

Figure 6.1a compares the performance of the currently applied model with the deep learning based approach. The DL-STPM-VPD model performs better according to the tournament ranking for 57% of the spare parts contained in the sample. This underpins the results from the significance test, neither of

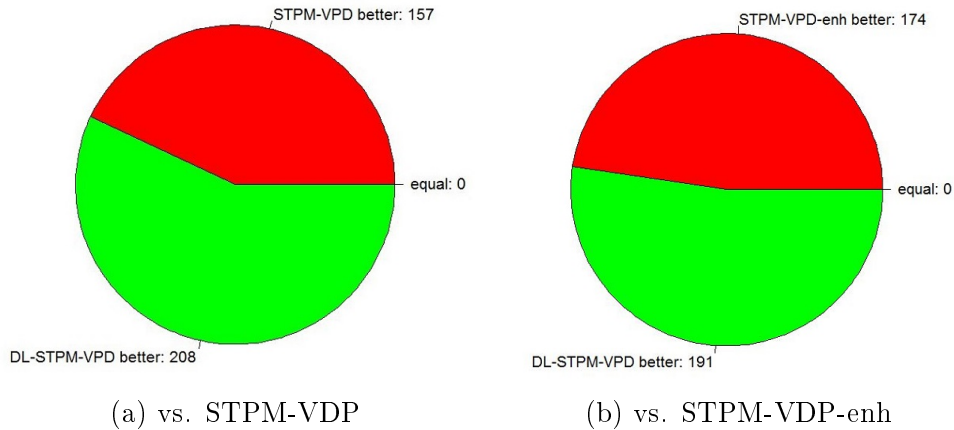


Figure 6.1.: Comparison against DL-STPM-VPD according to tournament ranking.

both can generalize all spare parts contained in the sample. Nonetheless, the proportion of the deep learning based approach is larger than the share of the currently applied model, what concludes that the proposed model improved the overall accuracy of the demand forecast. Furthermore, this encourages the analysis of the two resulting classes of spare parts, build by superior model performance in future work.

The results of the comparison of the enhanced version of the STPM-VPD model with the proposed deep learning approach is visualized in Figure 6.1b. The deep learning based model slightly performed better than the enhancement of the currently applied model by a proportion of 52% of the spare parts of the sample. On one side this confirms the achievements of the enhancements to the currently applied model by decreasing the proportion of parts the deep ANN performed better, compared to the previous comparison. On the other side it reduces the benefit of the proposed model because it only increased forecast accuracy for a few parts, compared to the enhanced version of the STPM-VPD model.

Figure 6.2 shows some exemplary diagrams, comparing the forecasts of the STPM-VPD and deep learning based model. Plot 6.2a and 6.2b present two parts the deep ANN outperformed the currently applied model. The proposed model was able to learn the relation between the demand and the VPD, resulting in a very accurate forecast, as long as vehicle data is available. As no further VPD is accessible the model starts to predict an average demand

value. This concludes that it is necessary that the VPD is available for the whole forecast horizon.

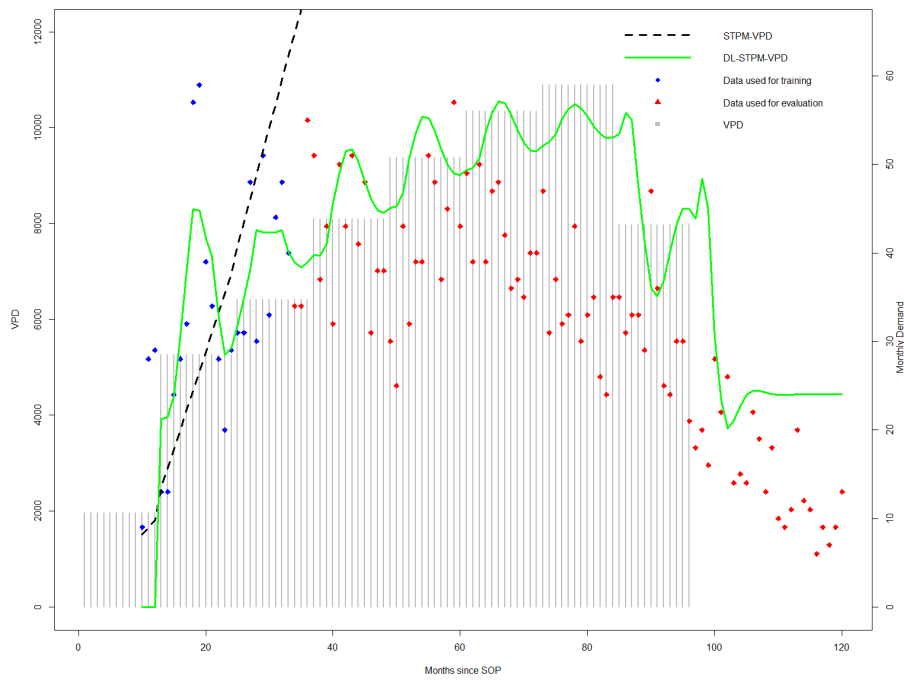
The Diagrams 6.2c and 6.2d show parts where the currently applied model outperformed the proposed approach. In the first case the deep ANN was not able to learn the relation of an increasing demand if the cumulative sum of vehicles grows, resulting in an underestimation of the real spare part demand. In the second case the missing VPD after a few time steps forced the deep model to rely on the demand data only, misconstruing the last months of training data and ignoring that no further vehicles are added to the market. This ends in a substantially overestimated spare part demand.

Figure 6.2e represents a case both compared models had difficulties to learn the demand pattern. The apparently not from market vanishing cars result in a steadily increasing demand. Both models were not able to capture this by the training data. Whereas in Plot 6.2f both models were able to learn the same demand increasing phenomena. A potential explanation could be the slightly stronger increasing demand within the training data in the latter case.

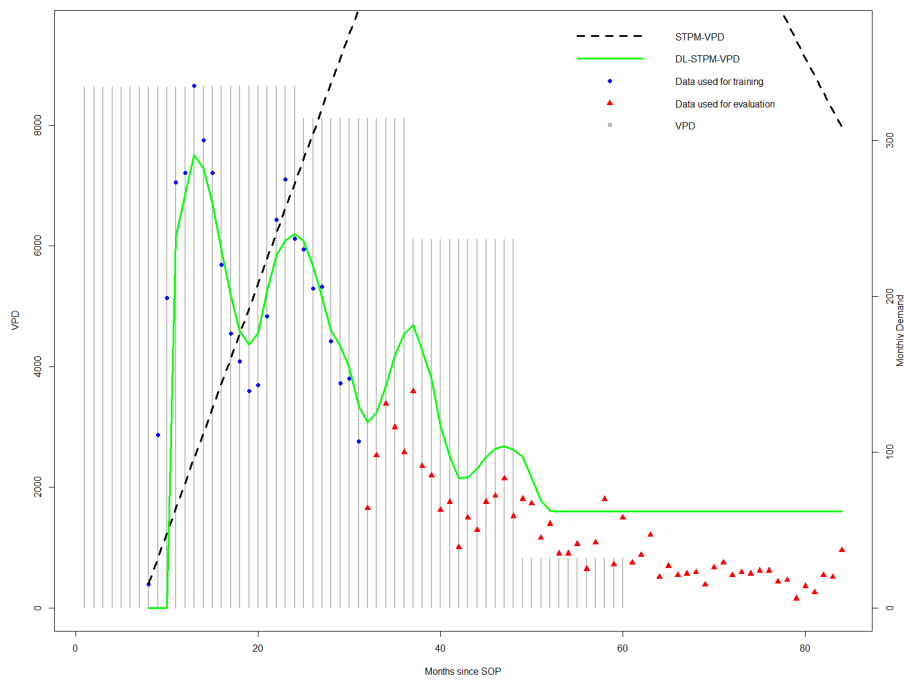
Figure 6.3 presents some exemplary spare part forecasts, comparing the STPM-VPD-enh and DL-STPM-VPD model. Diagrams 6.3a and 6.3b show parts the deep learning based approach performed better than the enhanced version of the currently applied model in terms of forecast accuracy. Either the STPM-VPD-enh model over- or underestimated the true demand. This may originate in a misleading vehicle depletion rate, learned by the regression model in both cases.

The Plots 6.3c and 6.3d are representatives of spare parts the enhanced currently applied model outperformed the deep ANN. In both cases the neural network was not able to learn the correct relation between the VPD, the vehicle depletion and the demand. This results in substantial over- or underestimation of the real spare part demand. Both visualized parts contradict the spare parts where the DL-STPM-VPD model performed better, because there is no obvious difference between all four spare parts. Nevertheless, the reasons should be investigated in a future study.

Finally, Figure 6.3e shows a part both models having trouble to accurately predict the spare part demand. Neither the deep ANN, nor the STPM-VPD-enh model were able to detect the correct relations and patterns describing this part. Therefore both models underestimated the demand. Plot 6.3f represents



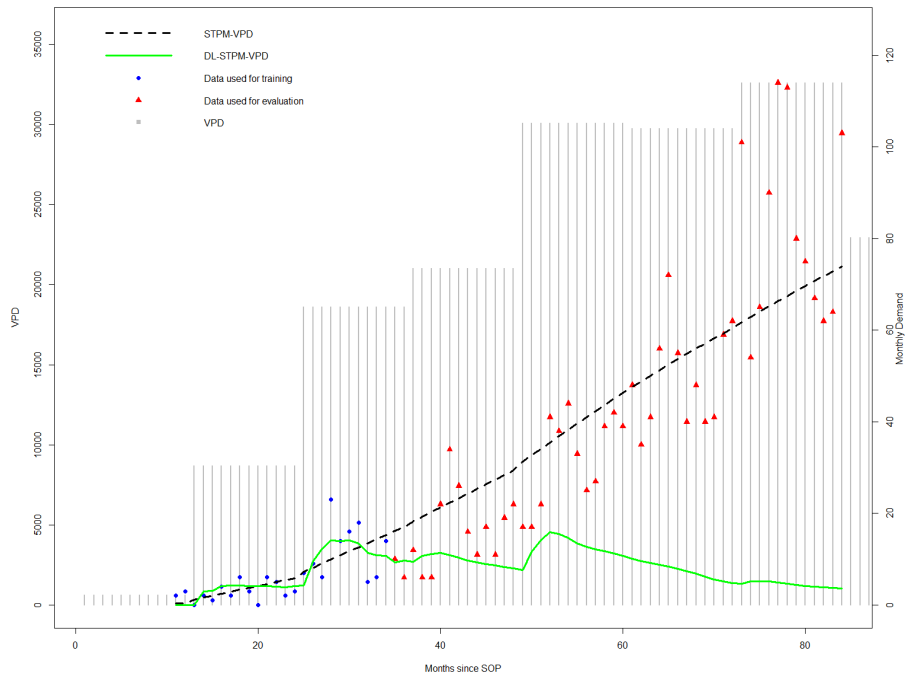
(a) DL-STPM-VPD better



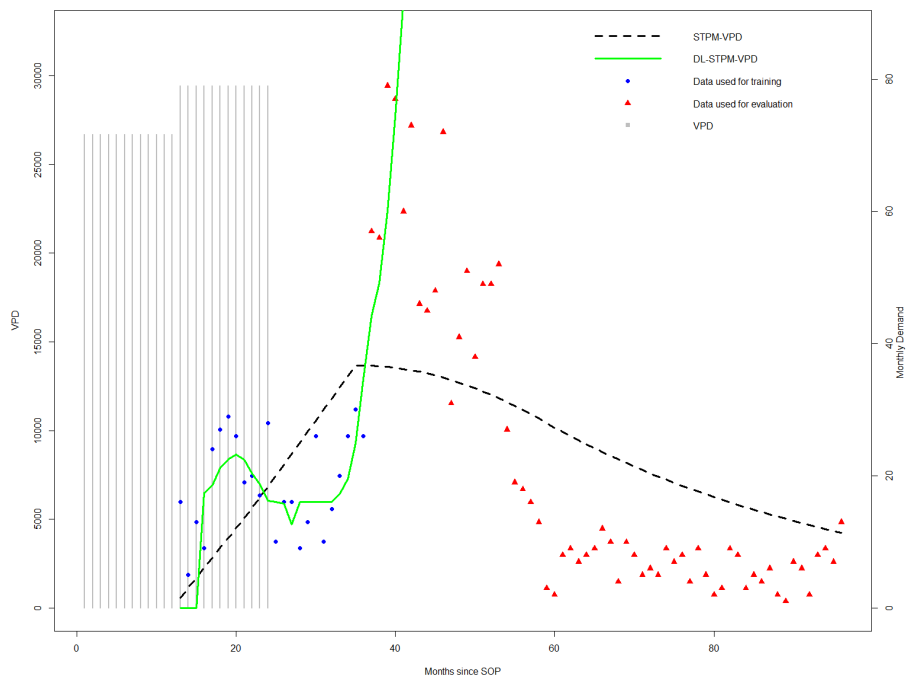
(b) DL-STPM-VPD better

Figure 6.2.: Example parts showing STPM-VDP and DL-STPM-VDP forecast.

6. Evaluation and Comparison of Proposed Models

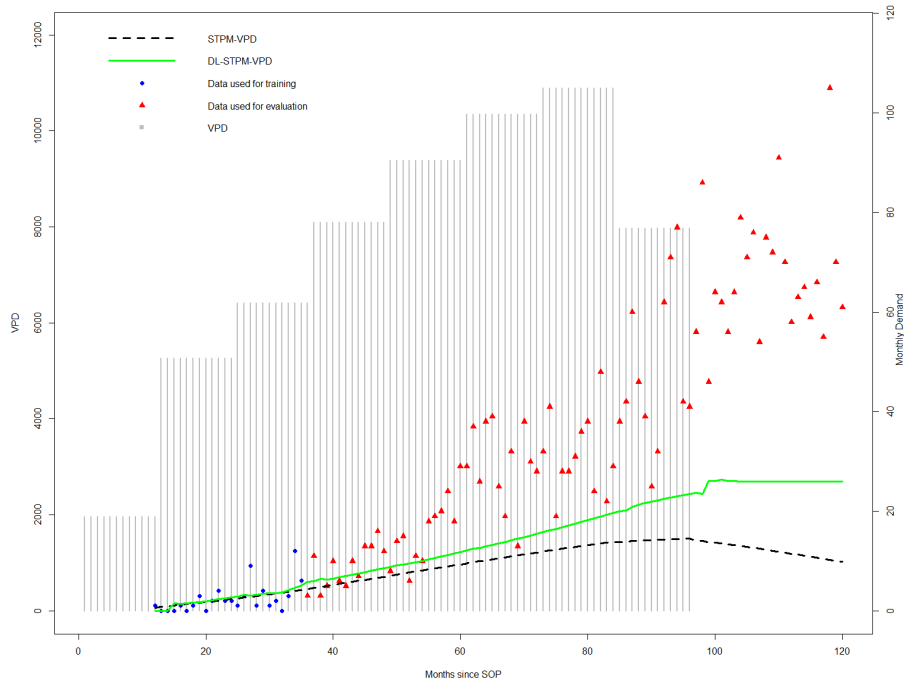


(c) STPM-VDP better

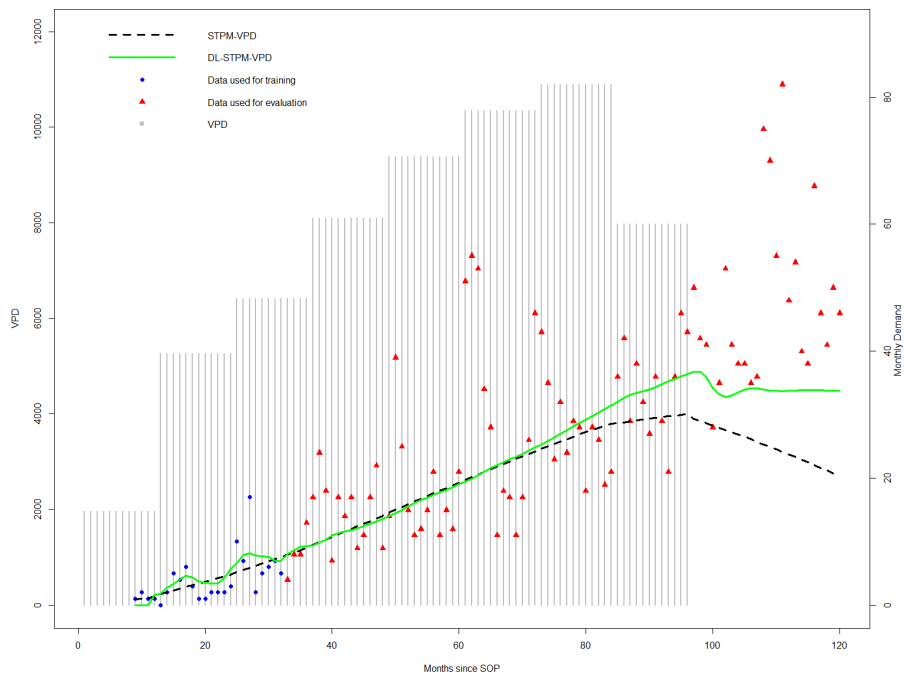


(d) STPM-VDP better

Figure 6.2.: Example parts showing STPM-VDP and DL-STPM-VDP forecast cont.



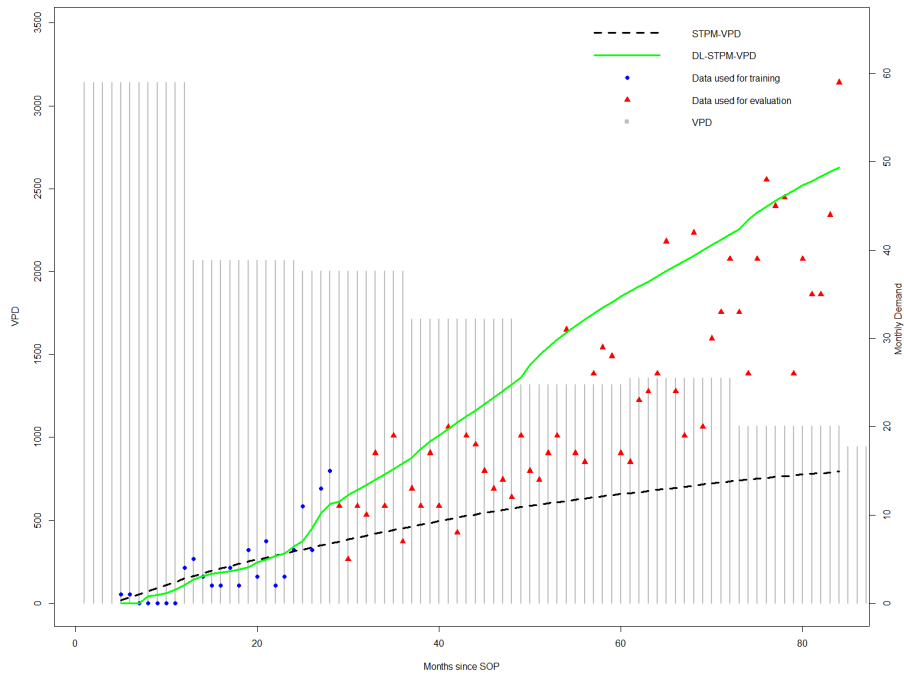
(e) both not satisfactory



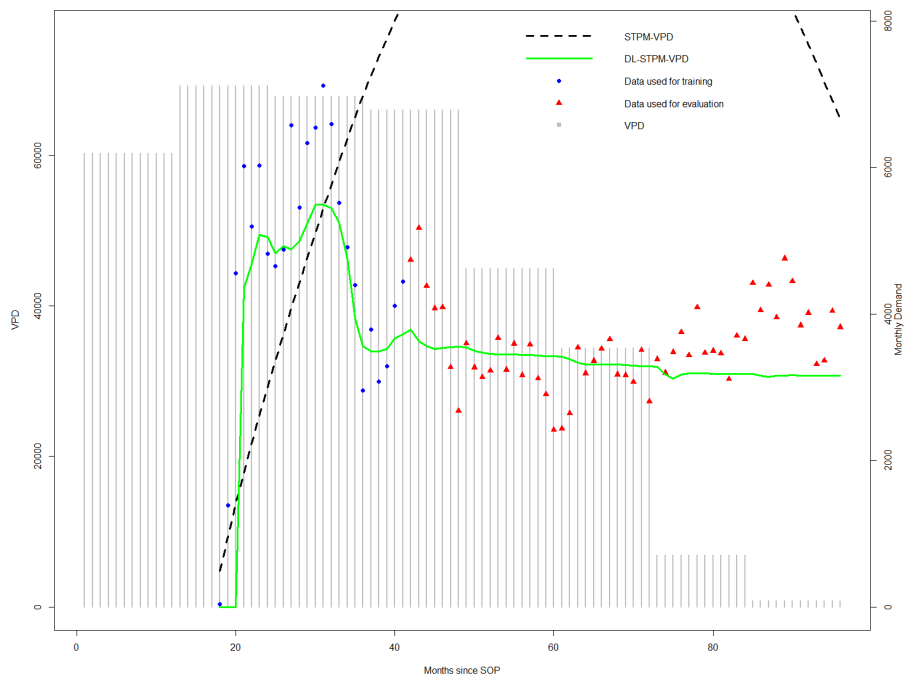
(f) both satisfactory

Figure 6.2.: Example parts showing STPM-VDP and DL-STPM-VDP forecast cont.

6. Evaluation and Comparison of Proposed Models

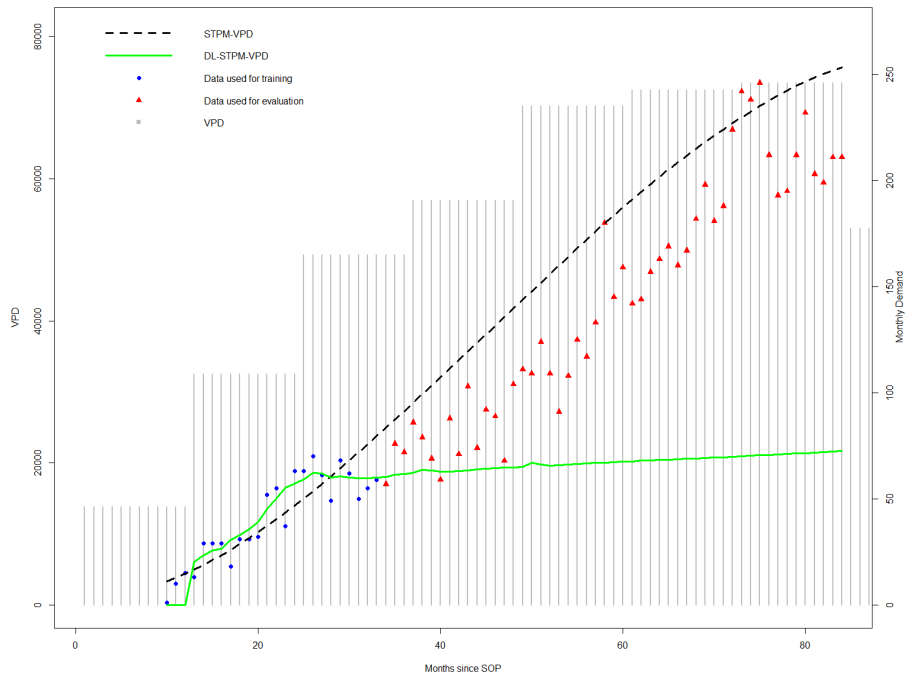


(a) DL-STPM-VPD better

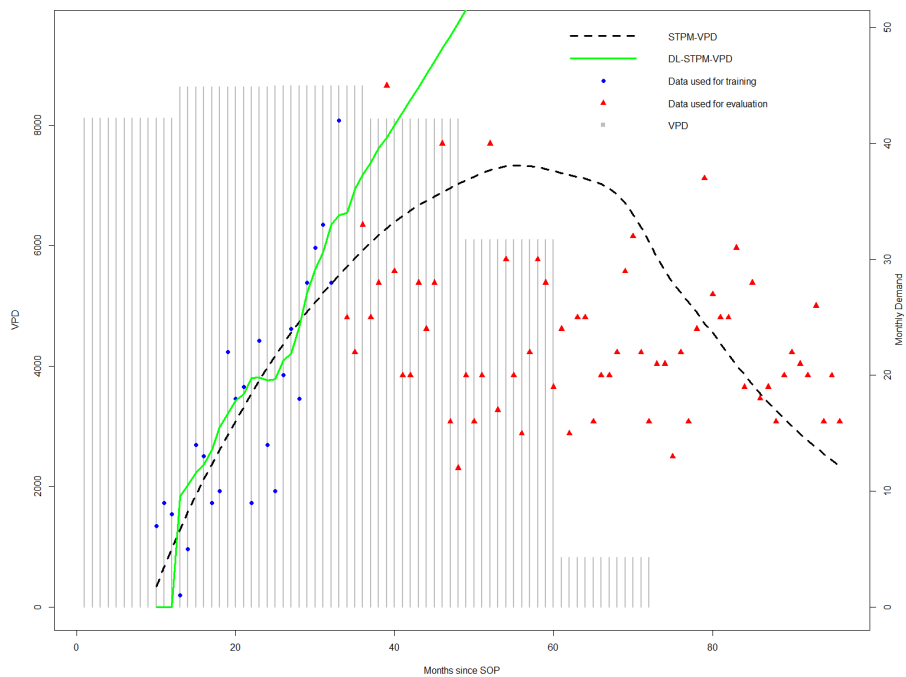


(b) DL-STPM-VPD better

Figure 6.3.: Example parts showing STPM-VPD-enh and DL-STPM-VPD forecast.



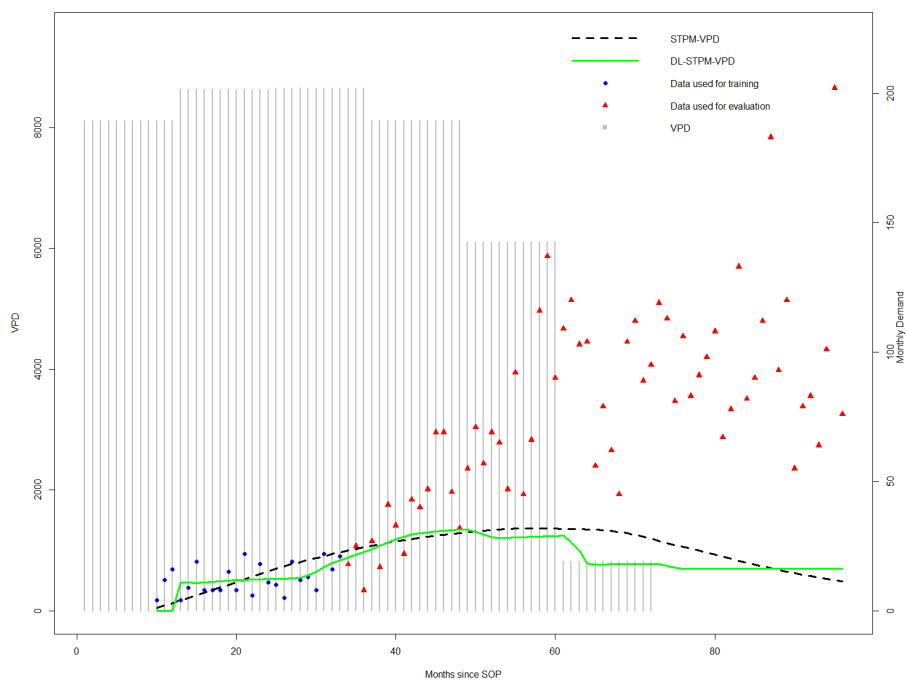
(c) STPM-VDP-enh better



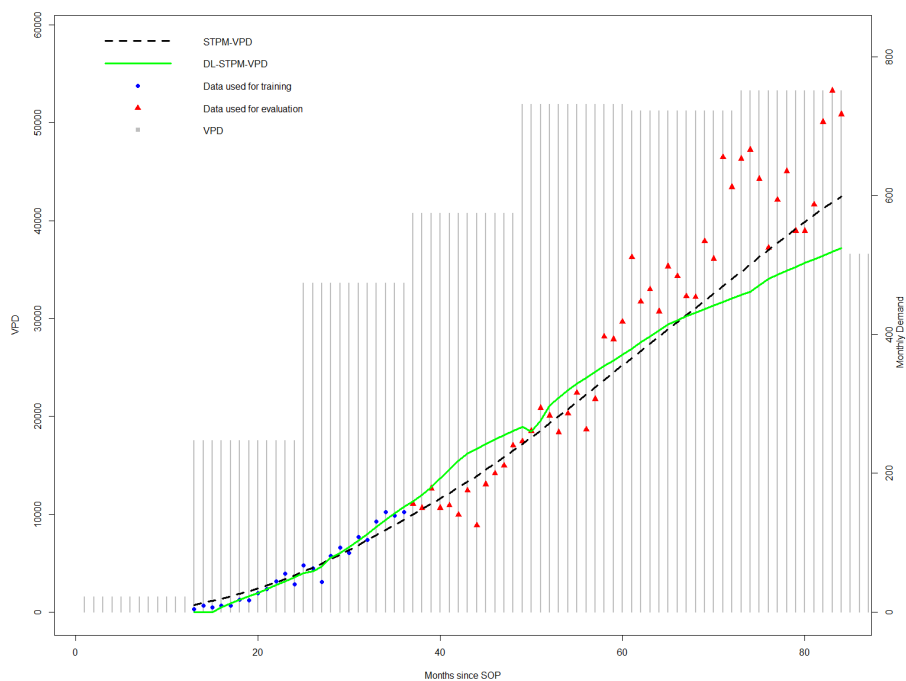
(d) STPM-VDP-enh better

Figure 6.3.: Example parts showing STPM-VDP-enh and DL-STPM-VDP forecast cont.

6. Evaluation and Comparison of Proposed Models



(e) both not satisfactory



(f) both satisfactory

Figure 6.3.: Example parts showing STPM-VDP-enh and DL-STPM-VDP forecast cont.

	Model	ψ_{bm}	ψ_{sp}
1	DL-STPM	353	168
2	STPM-enh	203	228
3	STPM	131	291

Table 6.2.: Significance ranking versus current model for DL-STPM.

a part predicted satisfactory by both models by learning the right relation between vehicles, vanishing vehicles and the occurring demand.

The visualized results support the already drawn conclusions, that there are categories of parts each model is superior to the other. To further increase the prediction accuracy these classes need to be analyzed and potential model optimization steps need to be identified.

6.2. DL-STPM

Table 6.2 summarizes the results of the significance test for the model without VPD. The deep learning based model clearly outperformed both other models in terms of ψ_{bm} . This result is confirmed by the ψ_{sp} values, DL-STPM achieving the lowest value, followed by STPM-enh and the currently applied model. The high ψ_{sp} scores indicate that the models usually performed significantly worse than the best model for a particular spare part. This concludes that each model has its group of parts it can handle better than the other models. These groups should be evaluated in a later study, whether the results could lead to new spare part classes, making use of all evaluated models. Nonetheless, the number of spare parts the deep learning based approach achieved a higher accuracy than the current model or its enhancement is greater than for the currently applied model, stating that the deep learning based approach achieved a higher forecast accuracy regarded for the whole sample as the STPM and STPM-enh model. This confirms the research question of this thesis in case of the univariate time series problem too.

Figure 6.4 shows the results if the currently applied model and the STPM-enh model are compared with the deep learning based model for the univariate time series based on the tournament ranking system. A model is again considered as better if it outperforms the compared one in at least two out of three evaluation

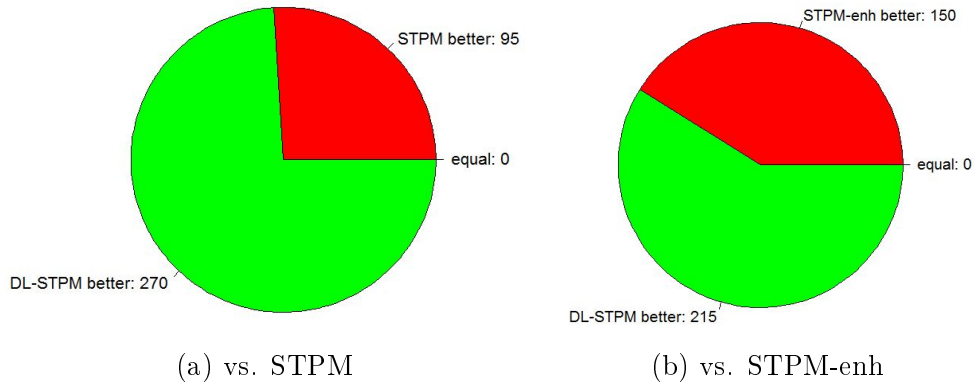


Figure 6.4.: Comparison against DL-STPM according to tournament ranking.

functions. Plot 6.4a visualizes the currently applied model versus the deep ANN. The DL-STPM model performed better for 74% of the evaluated spare parts. This is a clear improvement in terms of forecast accuracy, stating that the neural network approach is more suitable for the univariate time series problem than the currently applied model. Nevertheless, the resulting two classes of parts need to be investigated further to verify whether a solution containing both approaches is even more promising.

Image 6.4b represents the comparison of the enhanced version of the currently applied model with the deep learning based approach. The deep ANN model outperformed the proposed enhanced version of the currently applied model for 59% of the sampled spare parts. This leads to similar conclusions as for the multivariate time series. The proposed enhancements of the currently applied model are effective and reduce the superiority of the deep learning based model. Nonetheless, the DL-STPM model is still noticeable increasing the forecast accuracy compared to the STPM-enh model.

Figure 6.5 shows exemplary plots for selected spare parts, comparing the forecast of the currently applied model with the prediction of the proposed deep learning based approach. The Diagrams 6.5a and 6.5b visualize spare parts the deep ANN outperformed the STPM model. Whereas the DL-STPM model was able to capture the pattern of the time series data to some extent, the STPM model substantially overestimated the true demand.

The case of STPM outperforming the proposed deep learning based model is shown in Figure 6.5c and 6.5d. The parameters of the currently applied model were able to map the demand pattern based on the training data. The deep

ANN had problems to detect the relations within the data, which lead to a mean value, predicted few time steps after the end of training data, resulting in inaccurate estimations of the true demand.

Figure 6.5e represents a spare part both models were not able to satisfactory predict the future demand. Nevertheless, to predict the increasing demand pattern based on the available information in the training data is a tough task. Finally, Plot 6.5f shows a case both models produce similar results, that could be regarded as satisfactory based on the training input. It may be noted that in most cases either the one or the other model predicts the demand more or less correct. The case of both models doing a good job is rather seldom.

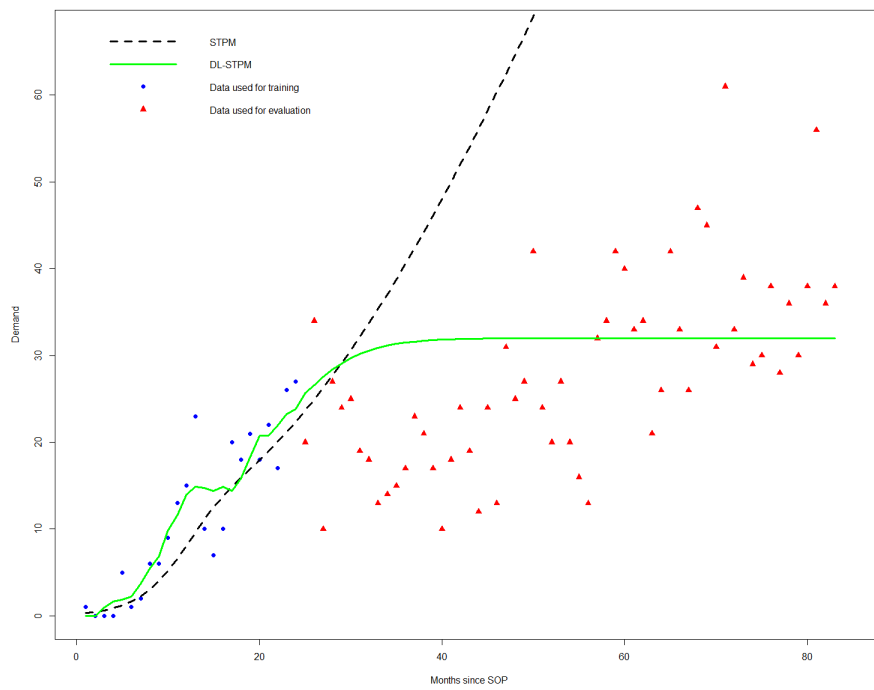
Image 6.6 visualizes the predictions of the STPM-enh and univariate deep ANN for some selected spare parts. The Plots 6.6a and 6.6b represent spare parts the proposed deep learning based approach achieved a higher accuracy than the enhanced version of the currently applied model. The DL-STPM model was able to learn the relations within the historic demand, even though the prediction accuracy decreases with increasing forecast horizon and becomes more and more an average demand like prediction. Nevertheless, both spare parts prove that the proposed approach could learn demand patterns from the few available data for training.

The Diagrams 6.6c and 6.6d present spare parts the STPM-enh model performed superior to the deep learning based approach. The latter one was able to fit a model to the training data, but the prediction performance decreases rapidly with increasing forecast horizon, resulting in not satisfactory demand predictions. The enhanced version of the currently applied model conversely predicted the demand with higher accuracy. This underpins the hypothesis of two classes of spare parts, same as for the multivariate time series problem.

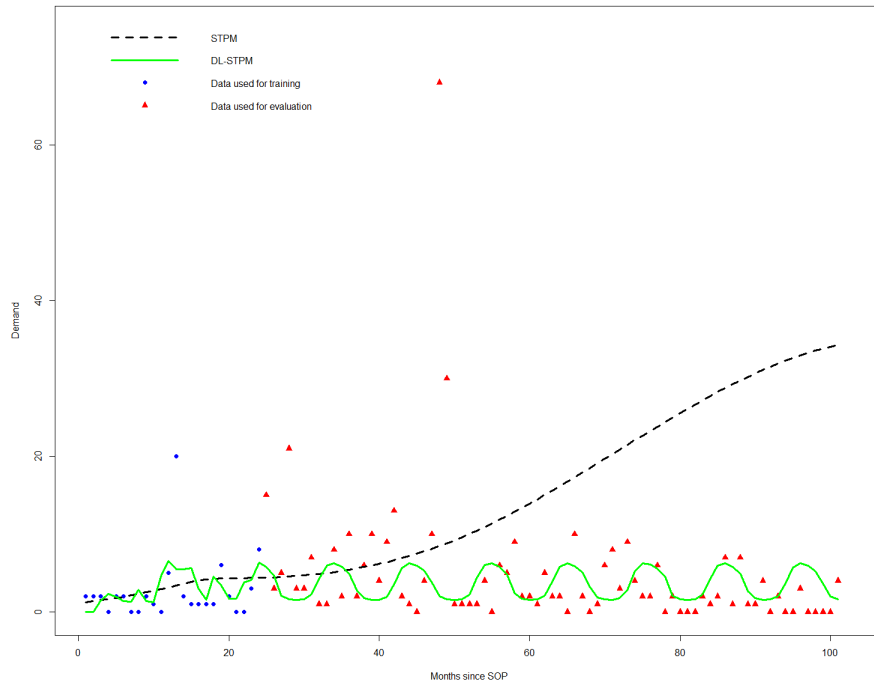
Plot 6.6e shows a part the deep ANN and the STPM-enh model performed rather bad in terms of prediction accuracy. Both models can deal well with the training data but cannot follow the upward trend of the demand curve. As sated earlier, this is a tough task if this trend was not indicated by the training data. Last but not least, Figure 6.6f visualizes the predictions for a spare part both models could forecast with satisfactory accuracy.

The above discussed exemplary spare parts underline the toughness of the univariate time series problem. It is difficult to accurately predict the future demand with that few information available for model training. In case the

6. Evaluation and Comparison of Proposed Models

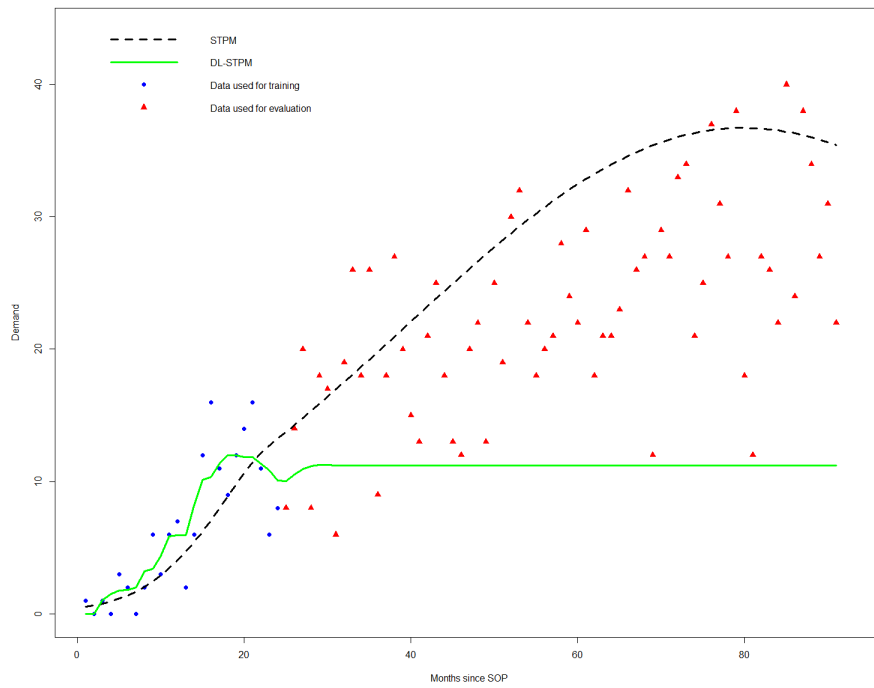


(a) DL-STPM better

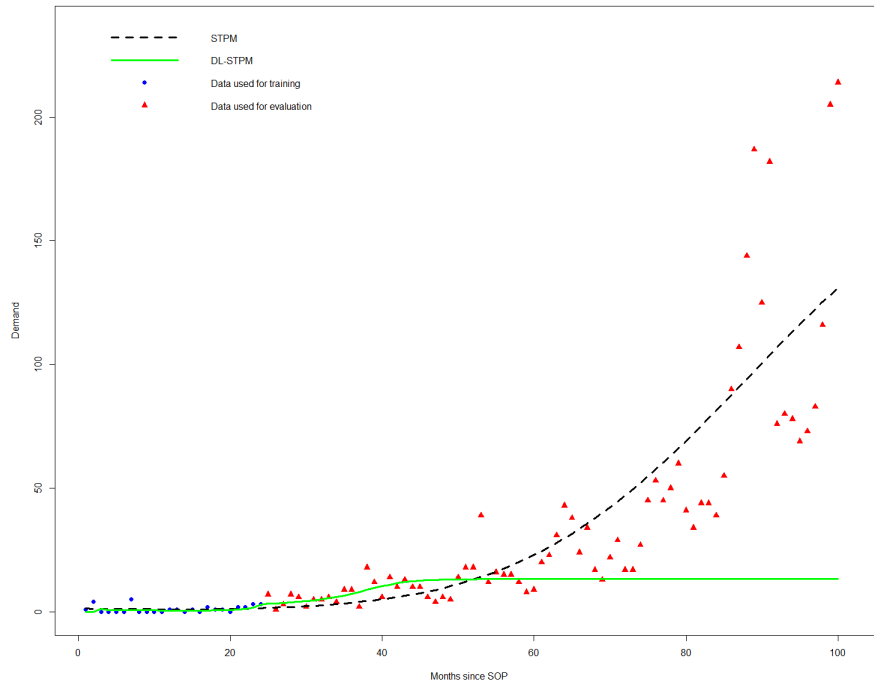


(b) DL-STPM better

Figure 6.5.: Example parts showing STPM and DL-STPM forecast.



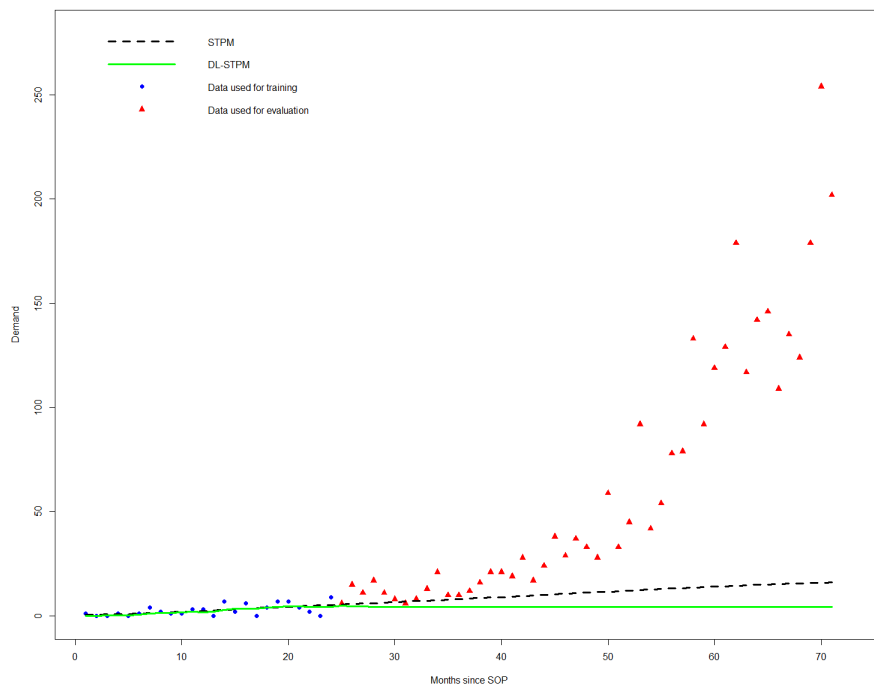
(c) STPM better



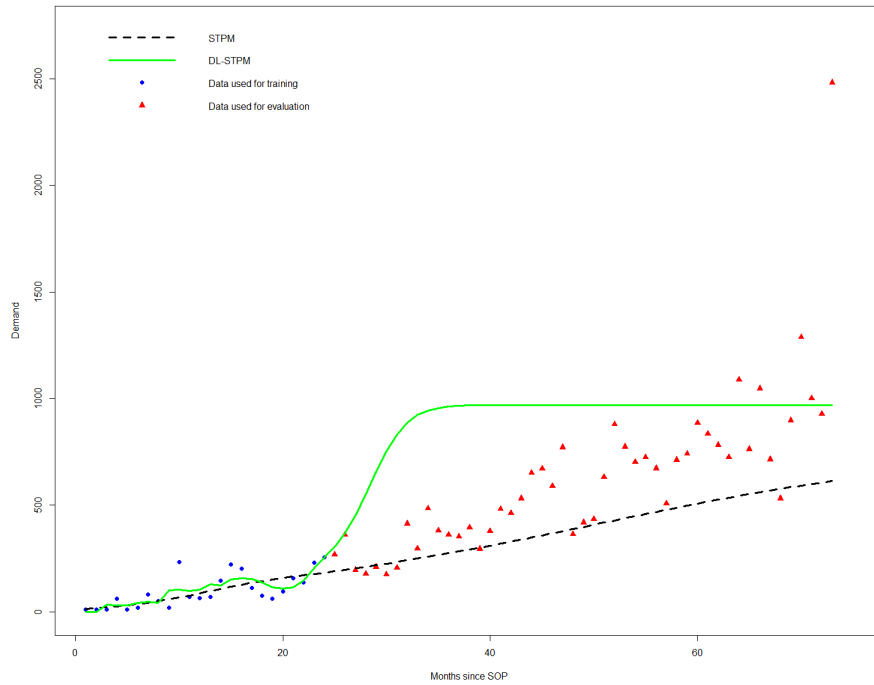
(d) STPM better

Figure 6.5.: Example parts showing STPM and DL-STPM forecast cont.

6. Evaluation and Comparison of Proposed Models

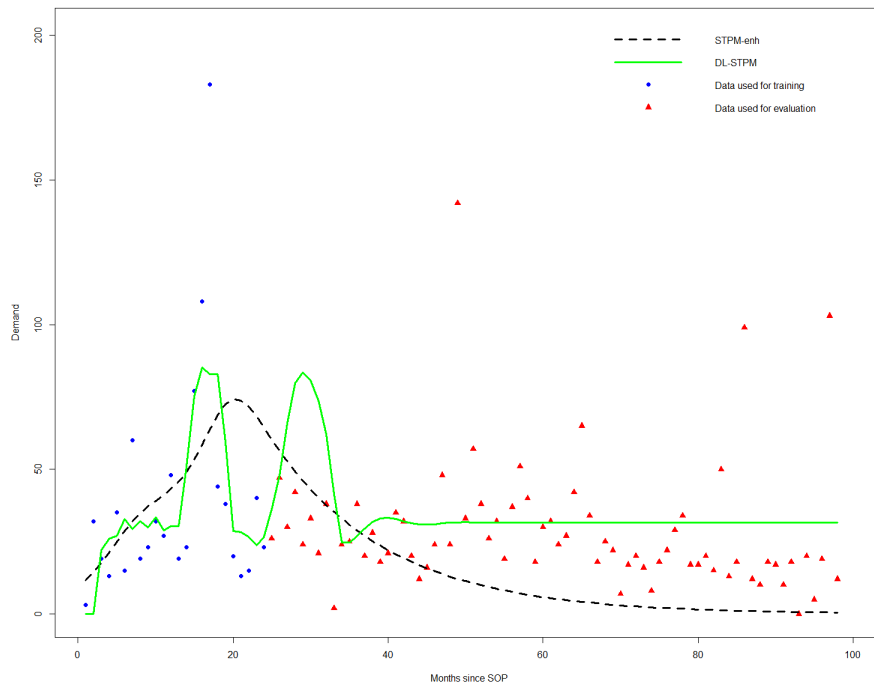


(e) both not satisfactory

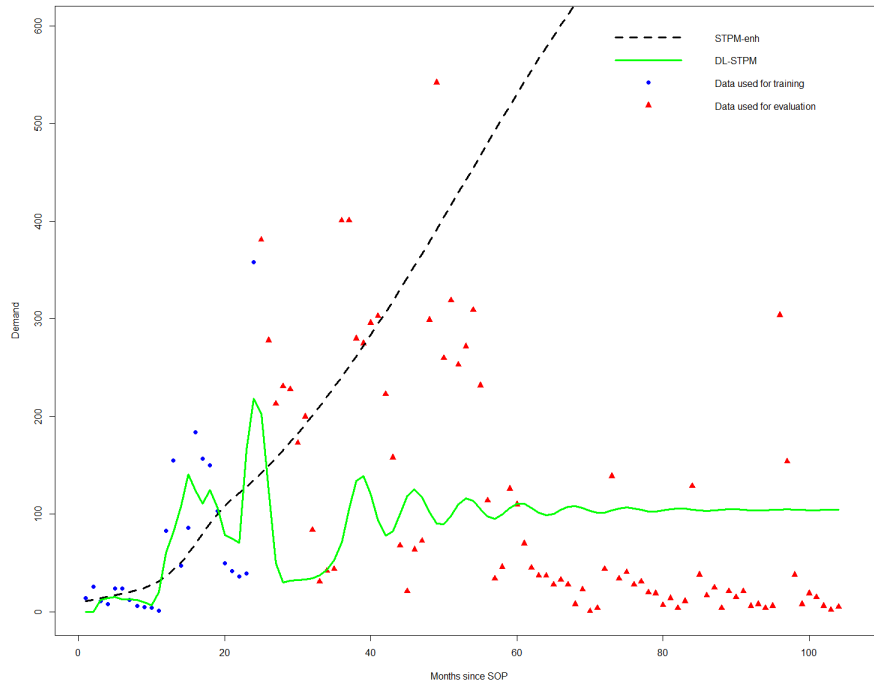


(f) both satisfactory

Figure 6.5.: Example parts showing STPM and DL-STPM forecast cont.



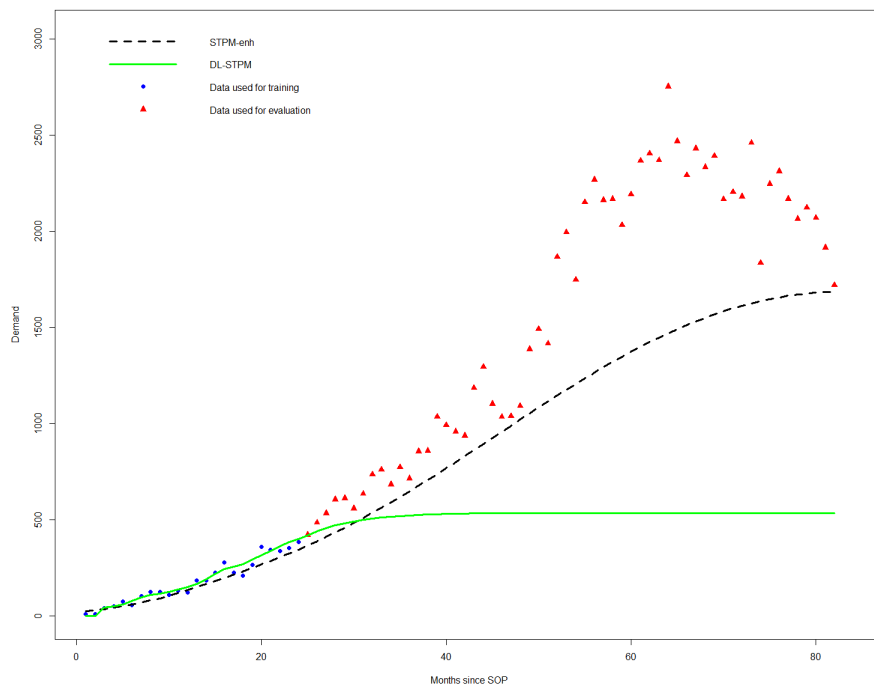
(a) DL-STPM better



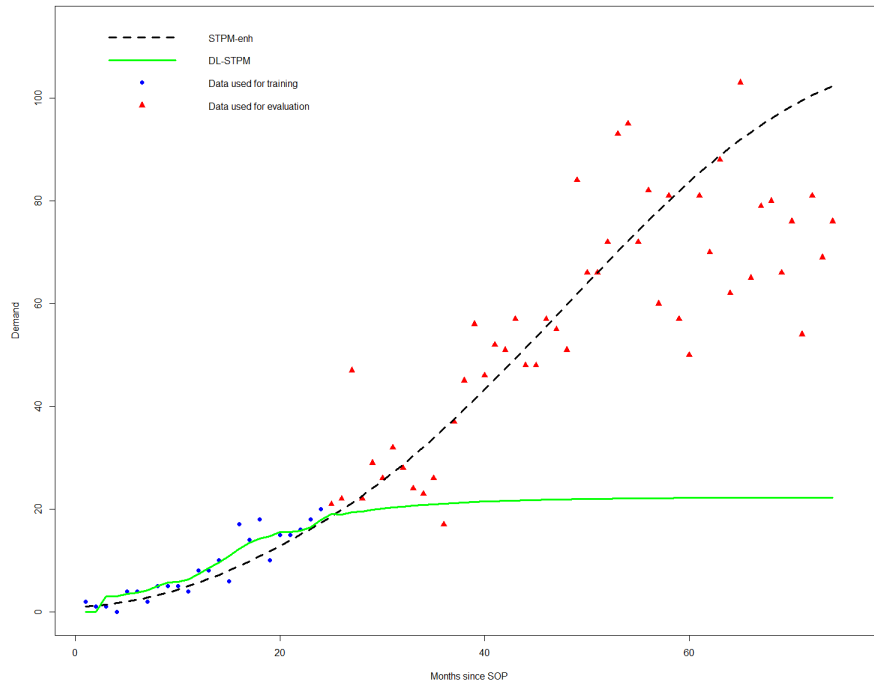
(b) DL-STPM better

Figure 6.6.: Example parts showing STPM-enh and DL-STPM forecast.

6. Evaluation and Comparison of Proposed Models

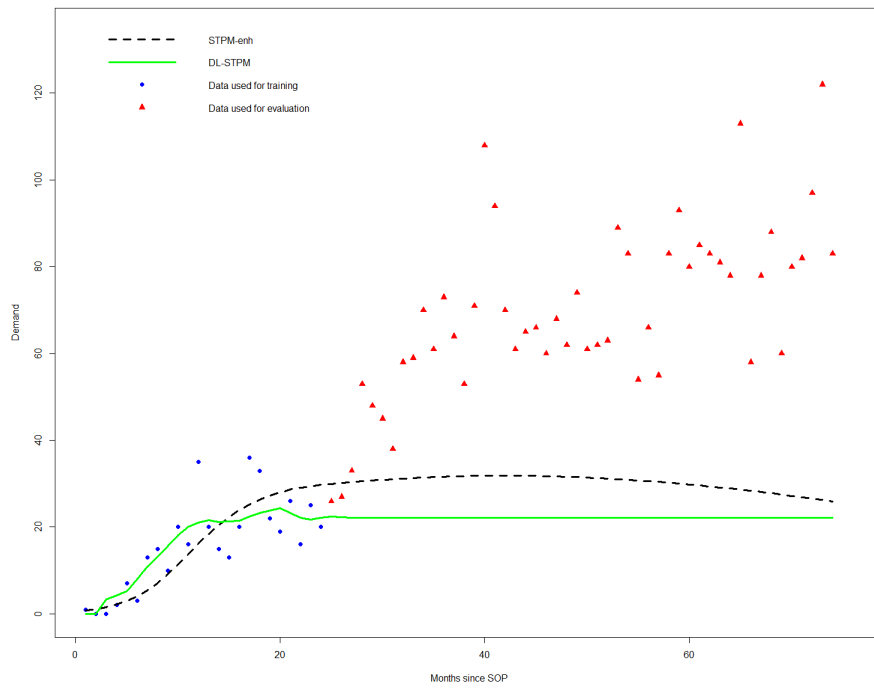


(c) STPM-enh better

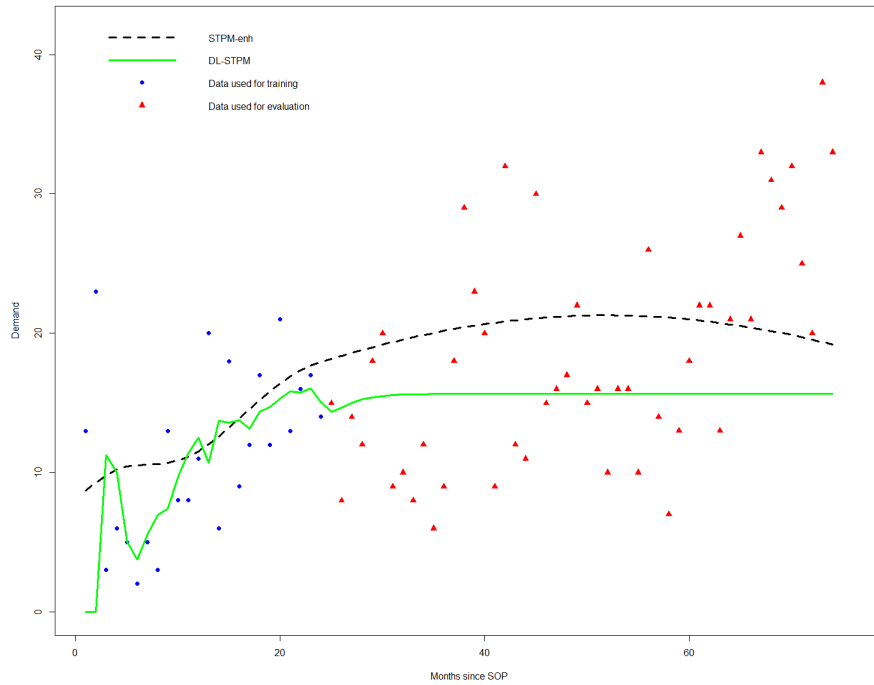


(d) STPM-enh better

Figure 6.6.: Example parts showing STPM-enh and DL-STPM forecast cont.



(e) both not satisfactory



(f) both satisfactory

Figure 6.6.: Example parts showing STPM-enh and DL-STPM forecast cont.

training data is much different from the later demand pattern the model often has no chance to place an accurate spare part demand prediction. Furthermore, the forecast horizon is shorter than for the multivariate time series data.

The final evaluation of the proposed deep learning model against the currently applied model and its suggested enhanced version showed that the deep ANN approach is superior. Therefore, the initial hypothesis of this thesis, if an Artificial Neural Network based prediction model forecasts the young fast-moving spare part demand with higher accuracy than the currently applied model, could be answered with yes. Even tough, this answer is stronger in case of the univariate model than in case of the multivariate, where only a slight performance increase was achieved. The results already showed promising research directions, to further increase the performance of the models.

7. Conclusion and Future Work

This work covered the development of a model for automotive spare part demand forecasting of young and fast-moving spare parts. The economic need of a model that optimizes the demand prediction and therefore supports the spare part management was discussed. Fundamental principles of the spare part management were introduced, and the characteristics of spare parts were analyzed. Furthermore, influence factors of the spare part demand were discussed to pave the way for a requirements driven analysis of possible approaches for demand prediction. To gain an overview an extensive literature review revealed models, that are applied for spare part demand forecasting, as well as models, that are suitable to meet the specified requirements. In parallel to the literature review the basic concepts of these models were introduced. According to state of the art research an Artificial Neural Network based approach was regarded as most promising. To form a basis for the evaluation of the model, the data available for evaluation were introduced and the currently for this task applied model, which should be outperformed in terms of forecast accuracy by the proposed approach was discussed. Furthermore, it was shown that enhancements of the currently applied model are possible but are elaborately and the gain is rather small.

Based on the requirements analysis and the results from literature review a deep learning based model, composed of densely connected, Elman and Long Short Term Memory layers was proposed in Chapter 5. Deep ANN are characterized by plenty of hyperparameters that could be tuned to improve forecasting performance. Due to the huge parameter space the optimal hyperparameters for the proposed model are experimentally derived and statistically evaluated on real world data, provided by a worldwide operating automotive company, by means of a sequential development process. The following hyperparameters were derived: the network architecture, the applied optimization algorithm and the related learning-rate, the activation function for each layer,

the size of the sliding window moved through training data, augmentation of the training data and the number of training epochs.

According to the developed hyperparameters the proposed model was compared with the currently applied model and its suggested enhanced version. The deep learning based model for automotive spare part demand forecasting was found to outperform both other tested approaches. The results were discussed in detail and weaknesses of the proposed model were identified, as well as possible solutions and starting points for further research.

7.1. Critical Summary

According to the results from the experimental evaluation of the proposed model, its superiority compared to the currently applied model and its enhancements is verified. This confirms reaching the primary target of this thesis of finding a model that could predict the spare part demand of young fast-moving spare parts with higher accuracy than the currently applied model. A limitation is, that this holds only if regarded for the whole sample of spare parts. There are spare parts for which the prediction accuracy was improved and there are parts for which the currently applied model is superior to the proposed approach. The deep ANN is regarded as superior because it outperforms the currently applied model for more than 50% of the spare parts, which in total is an improvement. Unfortunately, the applied evaluation measures only state whether a model outperformed the compared approach or not. For future research the evaluation measure should be changed in a way, that statements about the margin of enhancement are possible.

Furthermore, there have been some flaws identified. The applied significance evaluation measure introduced a bias to the derivation of the deep learning based approach. It prefers models that were found to be the best according to the tournament ranking system for the largest number of spare parts of the evaluated sample over models that performed satisfactorily, but were not the best on most of the spare parts. The bias holds for ψ_{bm} , counting only the number of significantly worse models for the best model of a spare part, as well as for ψ_{sp} , creating better results if a model was more often the reference vector. This bias prefers overfitting over generalization. Even though the likelihood of overfitting is rather low for the current scenario because of the few training

data that could be overfitted. Nevertheless, this should be kept track of. To overcome this bias an evaluation, significantly comparing all models for all parts, could be applied.

Continuously, a different order of hyperparameter derivation could have led to other results. In literature there exist no best practice in which order a deep ANN should be tuned. There are only recommendations, based on the expected effects of the hyperparameters, nonetheless these differ from task to task. Based on the experiences from this work, the order of hyperparameters derivations could be changed. The format of the input data, influenced by the data augmentation and the sliding window should be incorporated to the architecture design, because the architecture and the input are heavily related. Unfortunately, this increases the computational complexity of architecture derivation, which is expensive anyway.

An extension of the solution space regarding the hyperparameters is also recommended. Due to computational limitations the hyperparameters needed to be constrained for this study. This restriction should be reduced, and a wider parameter space evaluated. This implies a less restricted deep ANN, which could result in increased prediction accuracy. Nonetheless this produces a larger computational complexity for evaluation.

Last but not least, the sample selection should be mentioned. The sample size for hyperparameter evaluation allows a large margin of error, which in turn to some extent allows misleading conclusions. An increase of the sample size could reduce the margin of error but also requires a larger computational effort. Nevertheless, this is regarded as important because a larger sample probably better represents the true distribution, resulting in well-founded outcomes.

7.2. Outlook

Based on the results from the hyperparameter derivation and the comparison against the current model further research possibilities were identified. There seem to exist two classes of spare parts, one the deep ANN is superior and one the currently applied approach, or rather the enhanced version of the current model outperforms the proposed model. These two classes should be analyzed, and the characteristics of the particular spare parts should be identified. This could lead to an approach, where the spare part data is split, and each model

is applied to the subset it is more suitable for. This would further increase the forecast performance by using the more appropriate model for each spare part. Nonetheless, identification of the selection criteria for each subset could become a tough task.

The proposed model could be supplemented by a plausibility check of the forecast, similar to the approach suggested for the currently applied model. If this forecast fails to verify the plausibility of the outcome of the deep learning based model, the prediction is repeated with the currently applied model or its enhancement. This could overcome spare parts the neural network was not able to learn the demand pattern but the currently applied model or its modification is able to. A combination of both models via a plausibility check can improve the performance for spare parts, either of both approaches is capable of satisfactory predictions but not for parts, both models fail to forecast with sufficient accuracy. Nevertheless, the derivation of the rules to define prediction plausibility represents a future research point.

Furthermore, literature review identified several promising approaches. These not yet covered models could be evaluated on their own or combined to an ensemble. The ensemble approach would benefit from the advantages of all models. Verification of possible approaches capable of increasing the forecast performance compared to the currently applied model and the proposed deep ANN could be a starting point for further research. The combination of these to an ensemble extends this idea, but it is questionable if the computational effort needed to obtain the final model is feasible and useful.

The proposed approach could be evaluated for other spare part classes. It may be an interesting research point, how the deep architectures will perform on spare parts with a larger demand history and therefore more data available for model training. This could also increase the prediction accuracy for other classes of spare parts not covered yet.

Last but not least, changes to the input data should be evaluated more detailed. In the scope of this study a training period of 24 months was assumed. In reality this period ranges within 12 to 59 months, as of the selection criteria. The influence of different amounts of training data to the proposed model should be evaluated, to ensure its performance also under changing conditions. Continuously, the input data could be supplemented by expert knowledge, like expected spare part failure rates or usage statistics from the authorized workshops, to further support the model by finding the time series related patterns.

This additional information will also influence the needed amount of training data, as seen in the differences between the multivariate and univariate time series, used for model evaluation within this work. Pursuing, other theories, like phase space reconstruction of dynamical systems theory, could be evaluated to analyze if they could support the model by detecting the demand series underlying processes, to finally increase the spare part demand forecast accuracy.

Concluding, there are many directions for further research. This study revealed some new insights to the area of spare part demand forecasting, confirming that deep learning based models are capable of predicting future demand based on the historic spare part demand. This paves the way for future studies, possibly further increasing the prediction accuracy and therefore optimize the spare part management.

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A. Significance tables

A. Significance tables

Part	Best model	ϕ_{min}	$DL_{13-15-11}$	$DL_{13-14-11}$	$DL_{13-13-11}$	$DL_{13-12-11}$	$DL_{13-11-11}$	$DL_{13-10-11}$	$DL_{13-9-11}$	$DL_{13-8-11}$	$DL_{13-7-11}$	$DL_{13-6-11}$	$DL_{13-5-11}$	$DL_{13-4-11}$	$DL_{13-3-11}$	$DL_{13-2-11}$	$DL_{13-1-11}$		
Part 1	DRDRD-15-19-12-11-11	8	1.0000	0.1871	0.0482	0.0589	0.0714	0.0466	0.0244	0.2933	0.0698	0.0691	0.1733	0.0551	0.0136	0.4068	0.1479	0.0551	0.2981
Part 2	DLDRDL-26-19-19-13-15-11-9	40	0.0173	0.0057	0.4332	0.0106	0.0000	0.0000	0.0000	0.0002	0.0050	0.1520	0.0007	0.0136	0.0153	0.0063	0.4068	0.0551	0.2981
Part 3	DRDRD-30-19-14-5-5	10	0.0264	0.1027	0.1967	0.0649	0.4583	0.1027	0.0244	0.0254	0.0173	0.0007	0.1967	0.0218	0.2933	0.0466	0.1479	0.0551	0.2981
Part 4	DRDRD-18-17-12-11-11	12	0.0007	0.0075	0.0649	0.3064	0.0055	0.3481	0.5382	0.0106	0.5951	0.0305	0.8122	0.1688	0.3268	0.0466	0.1479	0.0551	0.2981
Part 5	DRD-15-13-5	43	0.0147	0.1186	0.1220	1.0000	0.0000	0.0516	0.0551	0.0038	0.0570	0.0141	0.0003	0.0001	0.0005	0.0000	0.1479	0.0551	0.2981
Part 6	DRD-15-13-5	5	1.0000	0.0218	0.3702	0.2224	0.8888	0.0833	0.8888	0.6935	0.9221	0.0406	0.7372	0.9443	0.4414	0.3064	0.8558	0.1645	0.2118
Part 7	DRD-15-18-7	19	0.0004	1.0000	0.0554	0.0913	0.8888	0.2681	0.8122	0.0254	0.0078	0.0082	0.1733	0.0043	0.0421	0.3338	0.2445	0.2224	0.0130
Part 8	DRDRD-15-17-14-13-7	5	0.1967	0.0450	0.0784	0.9888	0.9666	0.4250	0.4755	0.1027	0.0060	0.1479	0.1733	0.0153	0.3199	0.4250	0.3199	0.7267	0.0366
Part 9	DRDRD-30-17-16-7-9	23	0.9221	0.0017	0.0466	0.1364	0.7372	0.3553	0.1688	0.0533	0.0004	0.0001	0.0284	0.1027	0.0082	0.0048	0.0516	0.1440	0.0589
Part 10	DRDRD-18-9	46	0.0130	0.0700	0.1058	0.0450	0.0784	1.0000	0.0284	0.0180	0.0001	0.0002	0.0004	0.0002	0.0187	0.0000	0.0001	0.0026	0.0009
Part 11	DLDRDL-26-21-19-15-11-11-7	30	0.0000	0.0000	0.0153	0.0000	0.0078	0.0069	0.0001	0.0001	0.2503	0.0328	0.0244	0.0409	0.0516	0.0294	0.3627	0.0043	0.0014
Part 12	DLDRDL-26-19-19-13-15-11-9	2	0.8999	0.6687	0.2743	0.8778	0.3854	0.8068	0.6851	0.3338	0.9777	0.1220	0.2224	0.0546	0.1688	0.5063	0.2868	0.3854	0.0069
Part 13	DRDRD-15-17-14-13-7	3	0.3777	0.4583	0.4930	0.1186	0.4498	0.4169	0.0886	0.2067	0.4088	0.2388	0.0691	0.3131	0.4414	0.5569	0.1168	0.7906	0.1364
Part 14	DRD-18-9	1	0.0366	0.1967	0.4009	0.3409	0.3951	1.0000	0.3131	0.4668	0.2388	0.3064	0.7584	0.0628	0.7267	1.0000	0.1186	0.4498	0.4498
Part 15	DRDRD-15-19-12-11-11	14	0.0007	0.0392	0.0125	0.0088	0.0227	0.1918	0.0406	0.0141	0.2224	0.0227	0.5663	0.0008	0.1479	0.0736	0.2278	0.0125	0.0045
Part 16	DRDRD-30-11-5	45	0.0045	0.0001	0.0406	0.0000	0.0000	0.4009	0.0000	0.0000	0.1058	0.0015	0.0016	0.0031	0.3481	0.0040	0.0000	0.0002	0.0000
Part 17	DRDRD-18-17-12-11-11	18	0.0002	0.1327	0.1967	0.1778	0.4842	0.4028	0.0130	0.2561	0.0038	0.0589	0.1255	0.0284	0.4498	0.0450	0.3353	0.0136	0.7691
Part 18	DRD-18-13-11	15	0.9110	0.6851	1.0000	0.0736	0.0210	0.3777	0.1688	0.9591	0.6833	0.1027	0.0886	0.1121	0.0125	0.0435	0.1327	0.2805	0.5569
Part 19	DRD-30-11-5	43	0.3338	0.0466	0.0833	0.0833	0.2561	0.3854	0.5290	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Part 20	DRDRD-30-17-16-7-9	26	0.0691	0.0052	0.0005	0.0003	0.0000	0.6546	0.0608	0.1220	0.3627	0.0247	0.0628	0.0736	0.1645	0.2868	0.0736	0.0017	0.0097
Part 21	DLDRDL-30-17-13-9-13-13-11	27	0.0284	0.0628	0.2805	0.0714	0.0421	0.5461	0.0000	0.0608	0.1220	0.0247	0.0628	0.0736	0.1645	0.2868	0.0736	0.0017	0.0097
Part 22	DRD-18-15-11	30	0.0001	0.0392	0.0069	0.0069	0.0000	0.0018	0.0002	0.0218	0.3475	0.1645	1.0000	0.1153	0.5199	0.8999	0.0166	0.1479	0.0034
Part 23	DRDRD-15-19-12-13-11	13	0.0570	0.1440	0.3199	0.2118	0.8449	0.5569	0.2868	0.0366	0.0000	0.2278	0.0913	0.0085	0.0736	0.0028	0.6955	0.7798	0.3199
Part 25	DRD-18-9	43	0.6345	0.2621	0.0075	0.4755	0.0466	1.0000	0.6935	0.0022	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0043	0.0001
Part 26	DRDRD-22-19-8-13-7	12	0.3064	0.2681	0.3627	0.8778	0.6345	0.9666	0.4755	0.6445	0.6345	0.1561	0.1733	0.1364	0.0235	0.0353	0.1824	0.2933	0.8888
Part 27	DRDRD-22-11-8-7-11	17	0.0003	0.0005	0.0005	0.1440	0.0000	0.0006	0.0016	0.0001	0.2445	0.6955	0.6049	0.0202	0.4755	0.9110	0.0392	0.1089	0.9777
Part 28	DRDL-26-13-12-11-7	12	0.6345	0.7162	0.1967	0.8339	0.6147	0.9066	0.6935	0.6445	0.5382	0.6147	0.3409	0.6935	1.0000	0.0284	0.0886	0.2170	0.2933
Part 29	DRDRD-22-17-14-7-9	9	0.0000	0.0466	0.0969	0.0421	0.2445	0.0016	0.0085	0.0011	0.3481	0.4755	0.2278	0.2900	0.8014	0.6749	0.9554	0.0913	0.0000
Part 30	DRD-26-9-9	5	0.4250	0.7058	0.3854	0.3702	0.6851	0.6345	1.0000	0.7906	0.5108	0.9332	0.3481	0.5663	0.1824	0.9888	0.0202	0.1688	0.3199
Part 31	DRDRD-30-11-10-7-11	14	0.2998	0.7691	0.1967	0.7906	0.6747	0.4088	0.2743	0.0194	0.0000	0.0043	0.1027	0.0000	0.0450	0.0006	0.0218	0.0194	0.0013
Part 32	DRDRD-30-19-14-5-9	1	0.3553	0.3409	0.4088	0.9110	0.4088	0.5759	0.2805	0.3481	0.0670	0.4414	0.8014	0.7906	0.8014	0.8014	0.2743	0.2118	0.1778
Part 33	DRDRD-15-11-10-7-7	23	0.0392	0.0052	0.4930	0.4414	0.0101	0.0202	0.0940	0.0274	0.0024	0.0136	0.0533	0.0002	0.0036	0.0110	1.0000	0.4498	0.1327
Part 34	DRDRD-18-17-12-11-11	5	0.0000	0.0859	0.4088	0.5475	0.0366	0.0031	0.0859	0.0072	0.1290	0.1918	0.1220	0.3553	0.0998	0.5019	0.8014	0.8558	0.3777
Part 35	DRDRD-15-19-12-11-11	26	0.0002	0.0328	0.9443	0.0940	0.9221	0.8068	0.0940	0.0003	0.1089	0.0000	0.0013	0.0018	0.0120	0.0004	0.0305	0.0021	0.0886
Part 36	DRDRD-30-19-14-5-9	12	0.0808	0.0784	0.3064	0.2908	0.3854	0.6935	0.9332	0.3805	0.0317	0.8339	0.0670	0.4930	0.0913	0.4066	0.5759	0.3777	0.7162
Part 37	DRDRD-22-17-12-5-11	10	0.2170	0.4930	0.3931	0.0886	0.3109	0.6935	0.9332	0.3805	0.0317	0.8339	0.0670	0.4930	0.0913	0.4066	0.5759	0.3777	0.7162
Part 38	DRD-26-11-11	41	0.1918	0.4498	0.6749	0.1121	1.0000	0.5854	0.9888	0.0466	0.0019	0.0014	0.0002	0.0000	0.0006	0.0002	0.0187	0.0187	0.0055
Part 39	DRD-26-11-11	17	0.7162	0.6851	0.3131	0.2067	1.0000	0.8122	0.1918	0.1520	0.0166	0.0264	0.1153	0.2503	0.1027	0.0187	0.0187	0.0293	0.0659
Part 40	DLDRD-15-11-8-9-7	48	0.0012	0.0055	0.0628	0.0060	0.0060	0.0274	0.0072	0.0040	1.0000	0.0063	0.0008	0.0106	0.0000	0.0187	0.0003	0.0005	0.0001
ψ_{opt}		19	16	10	8	13	12	12	12	21	15	21	13	20	18	22	17	17	16

Table A.1.: Significance evaluation of 50 best architectures for DL-STPM-VPD.

Part	Best model	ψ_{best}	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11	DL-18-15-11
Part 1	DL-18-15-11	8	0.2561	0.0570	0.2808	0.4088	0.1688	1.0000	0.1645	0.0808	0.2903	0.0589	0.0353	0.1364	0.0147	0.1561	0.2118	0.0886	0.3481
Part 2	DL-18-15-11	40	0.0333	0.0110	0.0001	0.0003	0.0007	0.0027	0.0019	0.0019	0.0000	0.0000	0.0000	0.0019	0.0005	0.0000	0.0000	0.0001	0.0001
Part 3	DL-18-15-11	10	0.4583	0.8778	0.0392	0.2743	0.2008	0.3553	0.0379	0.3777	0.3777	0.4583	0.3199	0.0589	0.5475	0.3338	0.2067	0.6345	0.3064
Part 4	DL-18-15-11	12	0.0736	0.0516	0.0008	0.7267	0.4842	0.8378	0.4169	0.4332	0.2118	0.2224	1.0000	0.0829	0.0187	0.3561	0.0187	0.7162	0.3481
Part 5	DL-18-15-11	43	0.0015	0.0002	0.0023	0.0006	0.0274	0.0010	0.0125	0.0030	0.0000	0.0001	0.0048	0.0027	0.0011	0.0002	0.0531	0.0018	0.0005
Part 6	DL-18-15-11	5	0.1778	0.1153	0.1121	0.3702	0.0886	0.0036	0.0210	0.0549	0.0528	0.0528	0.1153	0.2906	0.0274	0.0345	0.0199	0.3827	
Part 7	DL-18-15-11	19	0.0670	0.0317	0.0066	0.1058	0.1153	0.1018	0.0688	0.1401	0.0736	0.3854	0.0379	0.2388	0.0153	0.5759	0.3199	0.0022	0.4332
Part 8	DL-18-15-11	5	0.0736	0.1169	0.4583	0.3481	1.0000	0.1121	0.3553	0.3409	0.1088	0.8122	0.8608	0.4414	0.7798	0.1733	0.2808	0.4169	0.5663
Part 9	DL-18-15-11	23	0.0333	0.0098	0.0024	0.0166	0.0017	0.2445	0.0194	0.0406	0.0649	0.0340	0.0913	0.8099	0.0098	0.1089	0.2743	0.0353	0.3481
Part 10	DL-18-15-11	46	0.0003	0.0166	0.0060	0.0012	0.0010	0.0008	0.0024	0.0097	0.0043	0.0002	0.0034	0.0264	0.0018	0.0015	0.0001	0.0120	0.0101
Part 11	DL-18-15-11	30	0.0125	0.0050	0.0106	0.0421	0.0033	0.0069	0.3199	0.0408	0.0021	0.0082	0.0089	0.2224	0.0482	0.0159	0.1121	0.0450	0.0340
Part 12	DL-18-15-11	2	0.9888	0.2118	0.2345	0.6145	0.2332	0.0068	0.0221	0.3732	0.0859	0.1364	0.0098	0.1561	0.3331	0.2883	0.2561	0.4169	0.2743
Part 13	DL-18-15-11	3	0.0421	0.3431	0.0340	0.0317	1.0000	0.0045	0.2017	0.0322	0.1733	0.0340	0.0561	0.2906	0.0039	0.2883	0.1186	0.1401	0.2881
Part 14	DL-18-15-11	1	0.1320	0.9666	0.2332	0.5739	1.0000	0.4668	0.2278	0.4668	0.1089	0.9777	0.4988	0.0245	0.1401	0.7601	0.9888	0.7162	0.3331
Part 15	DL-18-15-11	14	0.3651	0.9888	0.5382	0.1733	0.1479	1.0000	0.162	0.4921	0.0166	0.8122	0.9332	0.2503	0.7267	0.8778	0.3560	0.9221	0.8538
Part 16	DL-18-15-11	45	0.0055	0.0000	0.0063	0.0033	0.0010	0.0005	0.0005	0.0000	0.0001	0.0000	0.0001	0.0005	0.0002	0.0254	0.0022	0.0000	0.0055
Part 17	DL-18-15-11	18	0.8122	0.4482	0.3064	0.1561	0.3827	0.3409	0.1230	0.0670	0.1871	0.0317	1.0000	0.2332	0.2743	0.4842	0.2743	0.0066	0.3131
Part 18	DL-18-15-11	15	0.6546	0.0098	0.3067	0.0808	0.1327	0.0490	0.0913	0.0808	0.3199	0.0333	0.9827	0.0227	0.4350	0.5759	0.0218	0.1778	0.0736
Part 19	DL-18-15-11	43	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Part 20	DL-18-15-11	26	0.1027	0.0227	0.2361	0.0892	0.0045	0.2017	0.0072	0.0305	0.1441	0.8999	0.6589	0.2224	0.1220	0.7798	0.1121	0.1058	0.0340
Part 21	DL-18-15-11	27	0.0284	0.0110	0.0001	0.0043	0.0173	0.0691	0.0366	0.0421	0.0317	0.0097	0.0284	0.0075	0.0072	0.0136	0.1255	0.0048	0.0482
Part 22	DL-18-15-11	30	0.0069	0.0450	0.0153	0.1688	0.0366	0.0353	0.0030	0.0366	0.0389	0.0366	0.0366	0.0081	0.0254	0.1186	0.0406	0.0235	0.0649
Part 23	DL-18-15-11	32	0.0013	0.0153	0.0284	0.2118	0.0466	0.0166	0.0037	0.0101	0.0784	0.0406	0.0649	0.0001	0.0015	0.0015	0.0147	0.0027	0.0098
Part 24	DL-18-15-11	13	0.3338	0.9110	0.7267	0.2388	0.1440	0.7162	1.0000	0.0245	0.7058	0.6445	0.7006	0.8558	0.9221	0.6647	0.2868	0.8014	0.7384
Part 25	DL-18-15-11	43	0.0002	0.0499	0.0000	0.0570	0.0227	0.0027	0.0115	0.0689	0.0006	0.0006	0.0005	0.0000	0.0085	0.0180	0.0002	0.0040	0.0366
Part 26	DL-18-15-11	12	0.7306	0.0010	0.0008	0.3061	0.3777	0.2805	0.4009	0.1824	0.1683	0.6749	0.2170	0.0379	0.0840	0.6245	0.6749	1.0000	0.0649
Part 27	DL-18-15-11	17	0.5382	0.4498	0.0833	0.0008	0.2805	0.0015	0.0760	0.2561	0.0317	0.0366	0.1778	1.0000	0.1092	0.1561	0.0570	0.0110	0.0264
Part 28	DL-18-15-11	12	0.3199	0.1918	0.1733	0.4009	0.0294	0.1967	0.1186	0.7267	0.4668	0.0173	0.3199	0.0736	0.0091	0.1479	0.2332	0.1290	0.1918
Part 29	DL-18-15-11	9	0.2118	0.4842	0.6749	0.0482	0.0940	0.7601	0.9332	0.0588	0.1967	0.1689	0.0647	0.0013	0.3409	1.0000	0.2224	0.4842	0.1645
Part 30	DL-18-15-11	5	0.0628	0.2561	0.0345	0.2017	0.7058	0.4332	0.3331	0.5369	0.2278	0.3561	0.2278	0.0097	0.2332	0.4332	0.3777	0.4332	0.0940
Part 31	DL-18-15-11	14	0.2445	0.7058	0.2805	0.3200	0.7601	0.9118	1.0000	0.2881	0.3064	0.8230	0.0049	0.4755	0.6851	0.2067	0.6749	0.5108	0.6445
Part 32	DL-18-15-11	1	0.3854	0.0913	0.2170	0.3049	0.3827	0.1030	0.3854	0.4300	0.4009	0.2067	0.5759	0.3338	0.3331	0.9221	0.1733	0.6049	
Part 33	DL-18-15-11	23	0.1401	0.5369	0.1290	0.3777	0.9332	0.5108	0.4668	0.0049	0.3702	0.4380	0.1327	0.3702	0.3702	0.4482	0.0469	0.3854	0.0760
Part 34	DL-18-15-11	5	0.5475	0.6851	0.3369	0.8778	0.8122	0.3333	0.1255	0.2278	0.6667	0.2017	1.0000	0.5475	0.6245	0.5280	0.2278	0.3331	0.1255
Part 35	DL-18-15-11	26	0.2388	0.7584	0.1479	0.2868	1.0000	0.9110	0.0499	0.0499	0.3338	0.4021	0.0284	0.0003	0.2445	0.0305	0.0589	0.1153	0.0141
Part 36	DL-18-15-11	12	0.2332	0.0141	0.2081	0.2017	0.1733	0.3199	0.0328	0.3041	0.3338	0.0516	0.3409	0.8449	0.2445	0.3854	0.7601	0.4330	
Part 37	DL-18-15-11	40	0.3068	0.1658	0.5660	0.5108	0.3199	0.0499	0.5475	0.8888	0.1692	0.0392	0.0210	0.3338	1.0000	0.0066	0.2561	0.1824	0.4332
Part 38	DL-18-15-11	41	0.0018	0.0050	0.0010	0.0050	0.0020	0.0000	0.0028	0.0001	0.0017	0.0017	0.0005	0.0017	0.0082	0.0010	0.0015	0.0069	0.0038
Part 39	DL-18-15-11	17	0.1479	0.0736	0.3064	0.0808	0.0499	0.0379	0.3821	0.0435	0.0886	0.3651	0.0075	0.1089	0.0288	0.0551	0.0516	0.0298	0.3481
Part 40	DL-18-15-11	48	0.0001	0.0004	0.0003	0.0001	0.0027	0.0000	0.0000	0.0000	0.0008	0.0008	0.0003	0.0001	0.0001	0.0001	0.0005	0.0001	0.0001

Table A.1.: Significance evaluation of 50 best architectures for DL-STPM-VPD
cont.

A. Significance tables

Part	Best model	ϑ_{\min}	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000	00000000000000000000		
Part 1	DRDLD-5-47-8	34	1.0000	0.9693	0.0066	0.1440	0.0120	0.0016	0.0010	0.2908	0.0022	0.0052	0.0244	0.0003	0.0106	0.0000	0.0063	0.0028	0.0015	
Part 2	DRDLD-10-14-12-56-2-2	31	0.0000	0.4683	0.0004	0.7906	0.9777	0.9372	0.9332	0.2981	0.4988	0.0001	0.0889	0.0000	0.0194	0.0032	0.0833	0.0125	0.0328	
Part 3	DRDLD-5-8-9-6	35	0.871	1.0000	0.0000	0.1561	0.0028	0.1812	0.2621	0.2303	0.0000	0.0007	0.0001	0.0002	0.0001	0.0000	0.0004	0.0040	0.0000	
Part 4	DRDLD-5-8-11	12	0.0784	0.0089	0.4688	0.2908	0.8564	0.8004	0.2170	0.4755	1.0000	0.9435	0.9435	0.7038	0.3199	0.4985	0.1859	0.0172	0.0736	
Part 5	DRDLD-7-12-5-4	6	0.3927	0.6932	0.1186	0.8914	0.3661	1.0000	0.2908	0.0098	0.0859	0.1129	0.2933	0.7102	0.6749	0.3702	0.2989	0.2989	0.2302	
Part 6	DRDLD-7-11-6-5-10	6	0.2945	0.3663	0.4583	1.0000	0.1961	0.2743	0.9777	0.2278	0.2805	0.0940	0.1038	0.3117	0.1038	0.1479	0.3702	0.2627	0.3702	
Part 7	DRDLD-7-5-47-8	5	1.0000	0.8122	0.9221	0.8888	0.7058	0.7798	0.9777	0.5108	0.6245	0.0366	0.6698	0.7038	0.4414	0.0097	0.6698	0.7691	0.3927	
Part 8	DRDLD-5-9-12-7-6-3-6	3	0.4603	0.3854	0.2397	0.6936	0.4903	0.6922	1.0000	0.2334	0.4972	0.9829	0.8132	0.7191	0.8692	0.1431	0.3947	0.5143	0.4859	
Part 9	DRDLD-7-11-6-5-10	21	0.0015	0.7478	0.1918	1.0000	0.6851	0.1178	0.4930	0.2988	0.0019	0.0482	0.0075	0.0050	0.0003	0.0120	0.0130	0.0024	0.0101	
Part 10	DRDLD-5-47-8	29	1.0000	0.1918	0.0159	0.8668	0.1121	0.0499	0.3588	0.9666	0.0041	0.0006	0.0000	0.0000	0.0003	0.0000	0.0000	0.0000	0.0000	
Part 11	DRDLD-10-7-47-10	4	0.4009	0.8149	0.2278	0.2332	0.3131	0.2118	0.5768	0.4909	0.3702	0.4230	0.1918	0.3702	0.3854	0.3900	0.0305	0.8122	0.0000	
Part 12	DRDLD-15-9-10-3-10	25	0.0392	0.4250	0.0700	0.2224	0.0760	0.3657	0.9369	0.0305	0.1688	0.1367	0.0328	0.0886	0.0166	0.0021	0.6001	0.0392	0.0651	
Part 13	DRDLD-7-5-8-7-2-5-3	28	0.1602	0.0001	0.3268	0.8419	0.8122	0.4250	0.4842	0.0110	0.0345	0.0015	0.2388	0.0023	0.0010	0.0017	0.0690	0.0002	0.0003	
Part 14	DRDLD-7-12-5-10	38	0.1824	0.0628	0.0173	0.0516	1.0000	0.0003	0.0003	0.1327	0.0075	0.0284	0.0005	0.0000	0.0000	0.0009	0.0020	0.0006	0.0003	
Part 15	DRDLD-10-13-47-10	29	0.1290	0.2988	0.0001	0.1824	0.0097	0.6908	0.0097	0.0180	0.0097	0.0784	0.0007	0.0000	0.0106	0.0005	0.0041	0.0093	0.0006	
Part 16	DRDLD-10-5-8-3-8-3-4	8	0.0305	0.0784	0.8778	0.4566	0.6546	0.9584	0.9777	0.5769	0.7584	0.4088	0.7798	0.4095	0.6831	0.0745	0.1089	0.1778	0.3854	
Part 17	DRDLD-10-5-10-5-10	0	0.6245	0.3131	0.2445	0.1645	0.0714	0.5108	0.8909	1.0000	0.7372	0.2481	0.2967	0.2445	0.5019	0.0516	0.4009	0.8339	0.2933	
Part 18	DRDLD-5-8-9-6	15	0.6896	1.0000	0.5854	0.4755	0.8122	0.2777	0.6949	0.3481	0.3691	0.0292	0.0089	0.0340	0.0136	0.0120	0.0379	0.0075	0.0125	
Part 19	DRDLD-10-13-47-4-4	16	0.0001	0.0000	0.5882	0.8668	0.8449	0.3064	1.0000	0.0628	0.4755	0.3400	0.1027	0.2170	0.6847	0.1255	0.2332	0.8668	0.9366	
Part 20	DRDLD-10-7-47-10	24	0.0147	0.0001	0.8914	0.2067	0.8888	0.5108	0.5019	0.3777	0.4414	0.0063	0.0115	0.0292	0.0340	0.0000	0.0002	0.0024	0.0002	
Part 21	DRDLD-7-5-8-7-2-5-3	6	0.8339	0.1602	0.2332	0.0187	0.1401	0.1698	0.2128	0.2961	0.7798	0.8122	0.1871	0.4947	0.4498	0.4583	0.2910	0.2910	0.3927	
Part 22	DRDLD-15-13-10-9-10-5-6	26	0.9666	0.4109	0.0115	0.0305	0.0099	0.0045	0.0089	0.0082	0.0000	1.0000	0.0435	0.1645	0.1689	0.2968	0.2961	0.1689	0.0210	
Part 23	DRDLD-10-13-47-10	10	0.3987	0.8231	0.2910	0.1897	0.1773	0.0397	0.1512	0.0150	0.1209	0.0258	0.0659	0.3303	0.0617	0.4008	0.2968	0.2968	0.8796	
Part 24	DRDLD-5-8-11	38	0.0570	0.0097	0.0516	0.3131	0.3664	0.0017	0.1290	0.1384	0.0669	0.0002	0.0005	0.0000	0.0004	0.0003	0.0019	0.0012	0.0032	
Part 25	DRDLD-10-13-47-10	32	0.2917	0.6947	0.0001	0.1290	0.0167	0.1530	0.0141	0.0328	0.0069	0.0020	0.0000	0.0000	0.0001	0.0000	0.0052	0.0000	0.0000	
Part 26	DRDLD-13-9-10-5-10	37	0.0254	0.2743	0.0000	0.0007	0.4479	0.0159	0.0392	0.4688	0.0015	0.0069	0.0000	0.0000	0.0000	0.0000	0.0034	0.0002	0.0000	
Part 27	DRDLD-5-47-8	12	1.0000	0.4109	0.9332	0.0000	0.1733	0.0210	0.0284	0.0159	0.0055	0.4332	0.0017	0.4332	0.3777	0.8778	0.2361	0.1255	0.0000	
Part 28	DRDLD-13-11-8-3-2	23	0.0018	0.0000	0.2968	0.0072	0.0589	0.0218	0.0040	0.0147	0.9332	0.1355	0.7555	0.8538	0.5108	0.0469	0.1871	0.1871	0.9666	
Part 29	DRDLD-15-9-6-5-10-9-2	17	0.0028	0.0290	0.3777	0.0593	0.0066	0.0366	0.4109	0.0113	0.5475	0.0227	0.1871	0.2303	0.3443	0.8230	1.0000	0.6345	0.7384	
Part 30	DRDLD-15-7-10-7-6	0	0.3927	0.6945	0.3008	0.4414	0.7991	0.5250	0.5475	0.6861	0.4498	0.1440	0.0533	0.4414	0.4930	0.6345	0.6345	0.3298	0.3298	
Part 31	DRDLD-13-11-8-5-10-2-1	34	0.0027	0.0000	0.0305	0.0008	0.0014	0.0001	0.0013	0.0002	0.8339	0.0021	0.0516	0.0700	0.6147	0.1778	0.0210	0.0913	0.0499	
Part 32	DRDLD-15-9-12-7-10-2-3	20	0.0289	0.0042	0.6489	0.2908	0.4019	0.8143	0.5066	0.5084	0.1610	0.0629	0.1399	1.0000	0.6900	0.2213	0.0048	0.0228	0.7245	
Part 33	DRDLD-15-11-47-11-8-7-5	11	0.0136	0.0011	0.8888	0.8568	0.1479	0.2743	0.1401	0.8339	0.3131	1.0000	0.8561	0.4668	0.2933	0.0019	0.6851	0.5951	0.0760	
Part 34	DRDLD-15-5-6-11-4-2-6	26	0.0421	0.0093	0.9554	0.0001	0.0018	0.0002	0.2841	0.0013	0.1824	0.1401	1.0000	0.0700	0.2988	0.1479	0.4930	0.6647	0.2933	
Part 35	DRDLD-15-5-6-11-4-2-6	19	0.0187	0.0489	0.8888	0.0069	0.0034	0.0159	0.4414	0.0072	0.4414	0.8778	0.0516	0.4688	0.6345	0.2985	0.2170	0.1778	0.6628	
Part 36	DRDLD-5-7-10-7-10	27	0.0210	0.0023	0.0509	0.8908	0.0075	0.0024	0.0003	0.0093	0.0649	0.0069	0.2224	0.1645	0.0166	0.0466	0.0159	0.1587	0.0093	
Part 37	DRDLD-13-11-6-5-7-4	34	0.1645	0.0482	0.0005	0.0489	0.0031	0.1401	0.0133	0.1255	0.0305	0.0093	0.0013	0.0466	0.0004	0.0097	0.0194	0.0004	0.0033	
Part 38	DRDLD-10-7-47-10	4	0.4842	0.3772	0.1089	0.2968	0.1645	0.3303	0.1918	0.2968	0.0264	0.6851	0.4690	0.4842	0.1735	0.4755	0.4755	0.6033	0.6033	
Part 39	DRDLD-5-8-11	27	0.3338	0.8449	0.0082	0.4414	0.3290	0.0670	0.2131	0.0499	0.0264	0.0005	0.0001	0.0000	0.0004	0.0043	0.0012	0.0001	0.0001	
Part 40	DRDLD-5-8-11	34	0.6749	0.2968	0.0210	0.1401	0.2908	0.0210	0.0085	0.8449	0.0153	0.0000	0.0000	0.0004	0.0001	0.0003	0.0001	0.0000	0.0000	
		ϑ_{\min}	15	14	12	10	11	13	12	14	13	20	17	16	17	20	19	19	20	30

Table A.2.: Significance evaluation of 50 best architectures for DL-STPM cont.

A. Significance tables

Part	Best model	ψ_{hm}	ψ_{hp}	ψ_{hm}	ψ_{hp}	ψ_{hm}	ψ_{hp}	ψ_{hm}	ψ_{hp}	ψ_{hm}	ψ_{hp}	ψ_{hm}	ψ_{hp}	ψ_{hm}	ψ_{hp}	ψ_{hm}	ψ_{hp}
1	leakyRelU-SoftPlus-leakyRelU	6	0.0000	0.0000	0.0000	0.0000	0.0007	0.0024	0.0013	1.0000	0.0000	0.0013	0.3169				
2	RelU-leakyRelU-RelU	8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0069	0.0000	0.0000	0.0000	0.0013	0.0013				
3	leakyRelU-leakyRelU-leakyRelU	6	0.0045	0.0014	0.0091	0.0392	0.0998	0.0061	0.0466	0.0001	0.0001	1.0000	1.0000				
4	SoftPlus-RelU-SoftPlus	1	0.2743	1.0000	0.0781	1.0000	0.3382	0.3931	0.6851	0.1088	0.0888	0.0235	0.0235				
5	leakyRelU-leakyRelU-leakyRelU	5	0.0180	0.0075	0.3131	0.0227	0.0294	0.0040	0.1255	0.4088	1.0000	1.0000	1.0000				
6	leakyRelU-RelU-leakyRelU	7	0.3951	0.0002	0.0001	0.0034	0.0000	0.0000	1.0000	0.0101	0.0001	0.0001	0.0001				
7	RelU-RelU-RelU	6	1.0000	0.0482	1.1220	0.0038	0.0011	0.0244	0.0000	0.0000	0.0000	0.3131	0.3131				
8	leakyRelU-leakyRelU-leakyRelU	7	0.0072	0.0353	0.0533	0.0210	0.0050	0.0015	0.0030	0.0000	0.0000	1.0000	1.0000				
9	leakyRelU-SoftPlus-leakyRelU	1	0.4114	0.3730	0.0589	0.0998	0.0533	0.6851	0.0052	1.0000	0.0000	0.0736	0.0736				
10	leakyRelU-leakyRelU-leakyRelU	8	0.0000	0.0000	0.0006	0.0000	0.0001	0.0353	0.0000	0.0000	0.0000	1.0000	1.0000				
11	SoftPlus-leakyRelU-SoftPlus	1	0.7267	0.7478	0.2743	0.3409	0.2388	1.0000	0.5019	0.0450	0.0000	0.0681	0.0681				
12	leakyRelU-RelU-leakyRelU	3	0.8014	0.0511	1.0000	0.1645	0.6245	0.0066	1.0000	0.6147	0.0000	0.0274	0.0274				
13	RelU-leakyRelU-RelU	3	0.0691	0.6019	1.0000	0.4332	0.0808	0.0227	0.0173	0.0000	0.0000	0.0649	0.0649				
14	SoftPlus-SoftPlus-SoftPlus	6	0.0031	0.0421	0.0011	0.3777	1.0000	0.0284	0.0027	0.0000	0.0000	0.0316	0.0316				
15	SoftPlus-leakyRelU-SoftPlus	4	0.1918	0.6945	0.1320	0.0022	0.0000	1.0000	0.0125	0.0000	0.0000	0.7058	0.7058				
16	RelU-SoftPlus-RelU	0	0.2805	1.0000	0.1327	0.5199	0.0533	0.3702	0.1602	0.5200	0.3777	0.3777	0.3777				
17	RelU-RelU-RelU	7	1.0000	0.0002	0.0000	0.0106	0.8230	0.0000	0.4066	0.0254	0.0000	0.0000	0.0000				
18	leakyRelU-RelU-leakyRelU	6	0.2224	0.0005	0.0011	0.0000	0.0000	0.0000	1.0000	0.0000	0.0274	0.1186	0.1186				
19	SoftPlus-leakyRelU-SoftPlus	7	0.0000	0.0002	0.0141	0.0000	0.0010	1.0000	0.0000	0.0000	0.0000	0.1121	0.1121				
20	RelU-leakyRelU-RelU	7	0.0000	0.0210	1.0000	0.0004	0.0019	0.0141	0.0000	0.0000	0.0000	0.7906	0.7906				
21	leakyRelU-RelU-leakyRelU	7	0.0147	0.0000	0.0000	0.0000	0.0012	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000				
22	SoftPlus-leakyRelU-SoftPlus	3	0.4479	0.0069	0.0187	0.6147	0.6445	1.0000	0.0389	0.2445	0.0086	0.0089	0.0089				
23	leakyRelU-SoftPlus-leakyRelU	3	0.1255	0.0005	0.0001	0.1561	0.1186	0.0003	0.6049	1.0000	0.0000	0.3409	0.3409				
24	SoftPlus-leakyRelU-SoftPlus	2	0.4583	0.7691	0.2118	0.3931	0.2805	1.0000	0.0147	0.0000	0.0000	0.2968	0.2968				
25	leakyRelU-SoftPlus-leakyRelU	6	0.2017	0.0294	0.0000	0.0008	0.0001	0.0000	0.1121	1.0000	0.0000	0.0000	0.0000				
26	RelU-leakyRelU-RelU	3	0.3553	0.3131	1.0000	0.8099	0.2998	0.0147	0.0000	0.0000	0.0000	0.1401	0.1401				
27	SoftPlus-SoftPlus-SoftPlus	7	0.0000	0.0450	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0072	0.0000	0.0000				
28	RelU-RelU-RelU	6	1.0000	0.0069	0.0036	0.2388	0.8449	0.0000	0.0045	0.0000	0.0000	0.0159	0.0159				
29	leakyRelU-SoftPlus-leakyRelU	2	0.7058	0.1200	0.0003	0.0760	0.4842	0.0194	0.5475	1.0000	0.0000	0.2118	0.2118				
30	SoftPlus-RelU-SoftPlus	4	0.0008	0.0153	0.2388	1.0000	1.0000	0.1602	0.0000	0.0043	0.0000	0.0784	0.0784				
31	leakyRelU-SoftPlus-leakyRelU	6	0.0066	0.0000	0.0000	0.0008	0.6245	0.0000	0.3409	1.0000	0.0000	0.0097	0.0097				
32	SoftPlus-SoftPlus-SoftPlus	6	0.0007	0.0072	0.0048	0.8668	1.0000	0.6935	0.0000	0.0023	0.0000	0.0000	0.0000				
33	leakyRelU-RelU-leakyRelU	6	0.0760	0.0050	0.0000	0.0007	0.0000	0.0000	1.0000	0.5475	0.0000	0.0000	0.0000				
34	RelU-RelU-RelU	2	1.0000	0.6546	0.0784	0.8122	0.7267	0.0120	0.0000	0.1778	0.3064	0.3064	0.3064				
35	leakyRelU-RelU-leakyRelU	7	0.3200	0.0000	0.0000	0.0153	0.0340	0.0000	1.0000	0.0063	0.0000	0.0000	0.0000				
36	leakyRelU-SoftPlus-leakyRelU	8	0.0000	0.0000	0.0000	0.0012	0.0000	0.0001	0.0000	1.0000	0.0006	0.0006	0.0006				
37	RelU-leakyRelU-RelU	8	0.0000	0.0235	1.0000	0.0093	0.0000	0.4332	0.0000	0.0000	0.5475	0.5475	0.5475				
38	leakyRelU-RelU-leakyRelU	8	0.0011	0.0000	0.0000	0.0016	0.0000	0.0000	1.0000	0.0016	0.0000	0.0000	0.0000				
39	leakyRelU-RelU-leakyRelU	7	0.0736	0.0000	0.0001	0.0001	0.0003	0.0000	1.0000	0.0218	0.0000	0.0000	0.0000				
40	SoftPlus-SoftPlus-SoftPlus	6	0.0180	0.0366	0.0000	0.2621	1.0000	0.0003	0.0001	0.0000	0.0000	0.0001	0.0001				
		ψ_{hm}	18	29	21	20	28	21	20	25	21	25	18				

Table A.5.: Significance evaluation of Activation functions for DL-STPM-VPD.

A. Significance tables

Part	Best model	ψ_{bm}	w=2	w=3	w=4	w=5	w=6	w=7	w=8	w=9
1	w=3	6	0.0000	1.0000	0.1327	0.0000	0.0000	0.0000	0.0000	0.0000
2	w=2	4	1.0000	0.2332	0.5854	0.0101	0.7478	0.0000	0.0000	0.0000
3	w=9	4	0.0000	0.0000	0.0000	0.0366	0.2017	0.9443	0.5951	1.0000
4	w=2	3	1.0000	0.0516	0.8230	0.0317	0.0608	0.0194	0.0589	0.0028
5	w=3	7	0.0421	1.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
6	w=8	1	0.0691	0.2170	0.0499	0.7691	0.8122	0.4332	1.0000	0.4414
7	w=7	1	0.1058	0.1401	0.5854	0.2805	0.3409	1.0000	0.0050	0.4498
8	w=4	0	1.0000	0.5569	1.0000	0.8449	0.8014	0.1520	0.2621	0.1778
9	w=5	2	0.0038	0.9554	0.3931	1.0000	0.0317	0.1401	0.3777	0.1401
10	w=2	7	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
11	w=2	4	1.0000	0.1186	1.0000	0.0608	0.0000	0.0000	0.0000	0.0000
12	w=4	1	0.0147	0.6749	1.0000	0.4498	0.4583	0.8339	0.1255	0.1290
13	w=7	1	0.1058	0.0570	0.7478	0.5108	0.2681	1.0000	0.2868	0.0421
14	w=3	5	0.8339	1.0000	0.7162	0.0000	0.0000	0.0000	0.0000	0.0000
15	w=4	4	0.6445	0.0589	1.0000	0.4169	0.0001	0.0180	0.0008	0.0000
16	w=2	2	1.0000	0.2388	0.1520	0.1520	0.0760	0.1027	0.0194	0.0466
17	w=4	2	0.0406	0.9666	1.0000	0.0173	0.2170	0.7267	0.6749	0.1440
18	w=2	1	1.0000	0.9443	0.4009	0.5108	0.2332	0.0406	0.1440	0.3553
19	w=2	3	1.0000	0.9443	0.0913	0.6147	0.0691	0.0106	0.0000	0.0000
20	w=7	2	0.0005	0.0274	0.1824	0.3777	0.3409	1.0000	0.5759	0.5108
21	w=3	0	0.3268	1.0000	0.8558	0.3338	0.6445	0.1255	0.2621	0.1688
22	w=3	1	0.5569	1.0000	0.4498	0.8558	0.7162	0.8014	0.0482	0.6851
23	w=8	5	0.1645	0.0004	0.0000	0.0340	0.0482	0.0159	1.0000	0.9110
24	w=5	6	0.0000	0.0000	0.0000	1.0000	0.0305	0.0001	0.0516	0.0274
25	w=4	0	0.1602	0.3627	1.0000	0.6445	0.3481	0.7267	0.5569	0.4414
26	w=8	6	0.0000	0.0075	0.0015	0.0004	0.0040	0.0998	1.0000	0.0078
27	w=6	0	0.3854	0.5759	0.2388	0.4498	1.0000	0.7906	0.4332	0.5475
28	w=7	1	0.6546	0.0379	0.1520	0.8014	0.9666	1.0000	0.5199	0.8339
29	w=9	0	0.1520	0.9110	0.4169	0.7372	0.5569	0.4668	0.3931	1.0000
30	w=4	0	0.9666	0.8339	1.0000	0.6445	0.7584	0.5569	0.3338	0.4498
31	w=5	2	0.0048	0.0784	0.0340	1.0000	0.1778	0.1733	0.6546	0.1688
32	w=7	3	0.2681	0.4414	0.4009	0.3064	0.0284	1.0000	0.0001	0.0001
33	w=4	4	0.8339	0.6955	1.0000	0.0589	0.0055	0.0406	0.0063	0.0141
34	w=9	7	0.0001	0.0016	0.0000	0.0002	0.0000	0.0002	0.0060	1.0000
35	w=6	0	0.5569	0.0833	0.2561	0.4930	1.0000	0.8449	0.6851	0.2681
36	w=2	0	1.0000	0.8888	0.7478	0.6955	0.0940	0.1290	0.5019	0.5569
37	w=6	0	0.3931	0.7798	0.1121	0.1733	1.0000	0.7584	0.0714	1.0000
38	w=5	0	0.8230	0.5199	0.6851	1.0000	0.9554	0.5019	0.4755	0.4414
39	w=4	7	0.0005	0.0264	1.0000	0.0284	0.0294	0.0499	0.0115	0.0089
40	w=3	5	0.4755	1.0000	0.0218	0.0194	0.0014	0.0011	0.0392	0.3268
ψ_{sp}			12	9	10	13	15	16	16	16

Table A.7.: Significance evaluation of sliding window size for DL-STPM-VPD.

Part	Best model	ψ_{bm}	w=2	w=3	w=4	w=5	w=6	w=7	w=8	w=9
1	w=9	7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0499	1.0000
2	w=7	6	0.0005	0.0002	0.0033	0.0110	0.2017	1.0000	0.0000	0.0000
3	w=5	2	0.1918	0.1688	0.2445	1.0000	0.5382	0.1967	0.0034	0.0482
4	w=6	4	0.0000	0.0001	0.0005	0.0028	1.0000	0.7798	0.8778	0.2388
5	w=9	5	0.0000	0.0000	0.0000	0.0041	0.0649	0.0033	0.5199	1.0000
6	w=2	6	1.0000	0.0101	0.0714	0.0466	0.0000	0.0000	0.0000	0.0000
7	w=2	5	1.0000	0.6147	0.9221	0.0235	0.0130	0.0026	0.0021	0.0000
8	w=6	0	0.0551	0.2224	0.0589	0.3702	1.0000	0.3553	0.9221	0.6851
9	w=3	1	0.3199	1.0000	0.7906	0.4842	0.7162	0.1220	0.0628	0.0294
10	w=6	2	0.4414	0.8888	0.7584	0.8122	1.0000	0.8449	0.0015	0.0000
11	w=4	1	0.4009	0.4169	1.0000	0.4088	0.2278	0.1186	0.0649	0.0010
12	w=8	0	0.1871	0.1967	0.2998	0.9110	0.6345	0.2805	1.0000	0.6345
13	w=2	0	1.0000	0.9221	0.6049	0.7798	0.5475	0.3702	0.1778	0.2805
14	w=3	5	0.1520	1.0000	0.4414	0.0106	0.0328	0.0120	0.0038	0.0101
15	w=2	1	1.0000	0.5199	0.6647	0.3409	0.7267	0.6647	0.1561	0.0006
16	w=3	4	0.2561	1.0000	0.7691	0.6851	0.0366	0.0101	0.0012	0.0027
17	w=2	4	1.0000	0.5475	0.6749	0.0570	0.0141	0.0021	0.0000	0.0000
18	w=2	3	1.0000	0.4668	0.1290	0.2067	0.7058	0.0366	0.0000	0.0078
19	w=8	5	0.0000	0.0353	0.0153	0.0366	0.0110	0.5019	1.0000	0.4250
20	w=5	4	0.0005	0.0859	0.2681	1.0000	0.4088	0.0001	0.0000	0.0000
21	w=9	2	0.0001	0.0045	0.0833	0.9443	0.3481	0.7162	0.0969	1.0000
22	w=2	1	1.0000	0.0628	0.1255	0.2332	0.5290	0.1364	0.0093	0.1401
23	w=2	7	1.0000	0.0166	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
24	w=9	7	0.0180	0.0115	0.0340	0.0005	0.0018	0.0013	0.0392	1.0000
25	w=2	4	1.0000	0.3931	0.3931	0.9554	0.0000	0.0000	0.0000	0.0000
26	w=2	4	1.0000	0.7798	0.5108	0.5108	0.0043	0.0003	0.0000	0.0000
27	w=2	6	1.0000	0.0913	0.0210	0.0028	0.0000	0.0000	0.0000	0.0000
28	w=4	3	0.5569	0.2681	1.0000	1.0000	0.2681	0.0499	0.0075	0.0026
29	w=6	5	0.0340	0.0022	0.5569	0.1918	1.0000	0.0078	0.0000	0.0353
30	w=6	4	0.0002	0.0085	0.0082	0.0036	1.0000	0.7372	0.9666	0.3702
31	w=2	4	1.0000	0.1479	0.4842	0.7267	0.0130	0.0001	0.0000	0.0000
32	w=3	4	0.0166	1.0000	0.1089	0.2561	0.0913	0.0000	0.0000	0.0000
33	w=2	3	1.0000	0.0379	0.0886	0.2621	0.0736	0.0649	0.0000	0.0000
34	w=2	7	1.0000	0.0187	0.0136	0.0210	0.0000	0.0000	0.0000	0.0001
35	w=2	6	1.0000	0.0244	0.0833	0.0000	0.0000	0.0000	0.0000	0.0000
36	w=4	3	0.0533	0.1364	1.0000	0.4009	0.1733	0.0001	0.0000	0.0000
37	w=3	5	0.0153	1.0000	0.1290	0.0030	0.2332	0.0020	0.0001	0.0000
38	w=4	3	0.9443	0.0141	1.0000	0.5475	0.4009	0.1733	0.0052	0.0014
39	w=2	4	1.0000	0.8778	0.3131	0.0808	0.0055	0.0027	0.0004	0.0000
40	w=8	1	0.0001	0.0628	0.4498	0.5569	0.5854	0.8668	1.0000	0.5019
ψ_{sp}			13	15	10	15	16	23	28	28

Table A.8.: Significance evaluation of sliding window size for DL-STPM.

A. Significance tables

Part	Best model	ψ_{bm}	d=0	d=1	d=2
1	d=1	0	0.6049	1.0000	0.3064
2	d=0	2	1.0000	0.0000	0.0000
3	d=1	1	0.0010	1.0000	0.0670
4	d=0	1	1.0000	0.1186	0.0007
5	d=0	0	1.0000	0.6073	0.0846
6	d=0	1	1.0000	0.6851	0.0045
7	d=2	0	0.1479	0.6345	1.0000
8	d=1	0	0.4088	1.0000	0.0808
9	d=0	1	1.0000	0.2118	0.0000
10	d=1	1	0.0969	1.0000	0.0000
11	d=1	1	0.7691	1.0000	0.0027
12	d=0	2	1.0000	0.0023	0.0001
13	d=2	2	0.0101	0.0254	1.0000
14	d=0	0	1.0000	0.5382	0.3268
15	d=0	0	1.0000	0.0859	0.7478
16	d=0	1	1.0000	0.9332	0.0011
17	d=1	0	0.2278	1.0000	0.0998
18	d=1	1	0.0227	1.0000	0.2743
19	d=0	2	1.0000	0.0227	0.0000
20	d=0	2	1.0000	0.0421	0.0045
21	d=1	1	0.5854	1.0000	0.0041
22	d=2	2	0.0000	0.0033	1.0000
23	d=0	2	1.0000	0.0030	0.0235
24	d=1	0	0.1153	1.0000	0.0833
25	d=0	2	1.0000	0.0022	0.0004
26	d=2	0	0.6147	0.8122	1.0000
27	d=1	0	1.0000	1.0000	0.2278
28	d=0	1	1.0000	0.0305	0.0969
29	d=2	0	0.1089	0.4842	1.0000
30	d=2	0	0.0691	0.1602	1.0000
31	d=0	1	1.0000	0.2118	0.0000
32	d=0	2	1.0000	0.0450	0.0244
33	d=1	0	0.5475	1.0000	0.9443
34	d=2	1	0.0202	0.5569	1.0000
35	d=0	0	1.0000	0.7478	0.6647
36	d=2	1	0.0406	0.9888	1.0000
37	d=0	0	1.0000	0.7906	0.2445
38	d=0	2	1.0000	0.0499	0.0000
39	d=0	2	1.0000	0.0004	0.0000
40	d=1	0	0.8558	1.0000	0.0608
ψ_{sp}			6	12	17

Table A.9.: Significance evaluation of data augmentation for DL-STPM-VPD.

Part	Best model	ψ_{bm}	d=0	d=1	d=2
1	d=2	2	0.0004	0.0040	1.0000
2	d=1	0	0.1688	1.0000	0.2868
3	d=2	1	0.0002	0.0533	1.0000
4	d=2	1	0.0173	0.1255	1.0000
5	d=0	0	1.0000	0.8668	0.4088
6	d=0	2	1.0000	0.0000	0.0000
7	d=1	0	0.1089	1.0000	0.1327
8	d=2	2	0.0000	0.0000	1.0000
9	d=0	0	1.0000	0.1824	0.1290
10	d=0	1	1.0000	0.0066	0.5108
11	d=0	2	1.0000	0.0000	0.0000
12	d=0	2	1.0000	0.0063	0.0002
13	d=0	2	1.0000	0.0254	0.0435
14	d=0	0	1.0000	0.6955	0.7478
15	d=1	1	0.0421	1.0000	0.9110
16	d=2	1	0.0000	0.3199	1.0000
17	d=0	0	1.0000	0.1440	0.5663
18	d=2	1	0.0173	0.1778	1.0000
19	d=0	2	1.0000	0.0000	0.0000
20	d=2	0	0.8888	0.9332	1.0000
21	d=1	0	0.9666	1.0000	0.1255
22	d=2	0	0.3931	0.8668	1.0000
23	d=0	0	1.0000	0.1401	0.0784
24	d=0	0	1.0000	0.7478	0.0628
25	d=0	0	1.0000	0.4755	0.3064
26	d=0	0	1.0000	0.0940	0.5854
27	d=0	0	1.0000	0.3338	0.2332
28	d=2	0	0.2805	0.2278	1.0000
29	d=2	0	0.5951	0.8014	1.0000
30	d=1	0	0.2621	1.0000	0.5569
31	d=0	0	1.0000	0.2017	0.5951
32	d=1	0	0.6749	1.0000	0.8122
33	d=1	0	0.1733	1.0000	0.7162
34	d=0	0	1.0000	0.4842	0.2998
35	d=2	0	0.0628	0.1479	1.0000
36	d=2	1	0.0004	0.8888	1.0000
37	d=0	1	1.0000	0.0714	0.0254
38	d=2	0	0.6245	0.7691	1.0000
39	d=0	1	1.0000	0.0328	0.0516
40	d=0	2	1.0000	0.0002	0.0000
ψ_{sp}			8	10	7

Table A.10.: Significance evaluation of data augmentation for DL-STPM.

A. Significance tables

Part	Best model	ψ_{bm}	e=70	e=100	e=200	e=400	e=800
1	e=70	4	1.0000	0.0305	0.0000	0.0000	0.0028
2	e=70	1	1.0000	0.9888	0.0001	0.2681	0.1089
3	e=100	3	0.0886	1.0000	0.0435	0.0007	0.0063
4	e=200	0	0.8230	0.9110	1.0000	0.8558	0.6049
5	e=200	0	0.1121	0.5199	1.0000	0.0691	0.9666
6	e=200	1	0.7267	0.3131	1.0000	0.1186	0.0000
7	e=800	0	0.5759	0.8668	0.7058	0.9332	1.0000
8	e=70	2	1.0000	0.2224	0.4755	0.0000	0.0000
9	e=70	4	1.0000	0.0482	0.0002	0.0031	0.0000
10	e=100	0	0.8888	1.0000	0.3131	0.4583	0.0969
11	e=70	1	1.0000	0.6749	0.3338	0.0608	0.0024
12	e=400	0	0.3481	0.7372	0.9110	1.0000	0.0859
13	e=70	3	1.0000	0.0516	0.0008	0.0000	0.0000
14	e=70	0	1.0000	0.7162	0.7798	0.8449	0.6647
15	e=70	0	1.0000	0.9666	0.7906	0.8230	0.1918
16	e=100	3	0.3199	1.0000	0.0353	0.0000	0.0000
17	e=200	0	0.0466	0.4009	1.0000	0.3409	0.7058
18	e=200	0	0.0002	0.1290	1.0000	0.6245	0.9110
19	e=200	0	0.4169	0.2998	1.0000	0.8558	0.2868
20	e=800	0	0.0608	0.3931	0.3627	0.1733	1.0000
21	e=70	4	1.0000	0.0328	0.0353	0.0000	0.0000
22	e=400	1	0.3702	0.0244	0.7906	1.0000	0.0533
23	e=200	2	0.0670	0.2332	1.0000	0.0435	0.0030
24	e=70	1	1.0000	0.8449	0.4169	0.6445	0.0317
25	e=800	2	0.0089	0.0093	0.0120	0.1645	1.0000
26	e=70	4	1.0000	0.0120	0.0000	0.0024	0.0005
27	e=100	0	0.5019	1.0000	0.2017	0.3931	0.3199
28	e=200	2	0.1186	0.0969	1.0000	0.0066	0.0052
29	e=200	2	0.0023	0.0244	1.0000	0.1871	0.0009
30	e=800	0	0.0020	0.6147	0.1688	0.2998	1.0000
31	e=800	0	0.4009	0.4332	0.7058	0.7058	1.0000
32	e=70	2	1.0000	0.7058	0.4842	0.0001	0.0000
33	e=400	0	0.0353	0.5759	0.9332	1.0000	0.6546
34	e=400	0	0.8888	0.2445	0.1440	1.0000	0.0998
35	e=400	0	0.8014	0.9443	0.5854	1.0000	0.3409
36	e=200	0	0.9888	0.9777	1.0000	0.5663	0.8122
37	e=200	0	0.0379	0.4842	1.0000	0.2388	0.1918
38	e=400	1	0.0002	0.2388	0.0017	1.0000	0.5382
39	e=100	0	0.8122	1.0000	0.3777	0.8778	0.3338
40	e=70	4	1.0000	0.0089	0.0000	0.0000	0.0000
		ψ_{sp}	8	8	11	12	16

Table A.11.: Significance evaluation number of training epochs for DL-STPM-VPD.

Part	Best model	ψ_{bm}	e=70	e=100	e=200	e=400	e=800
1	e=800	4	0.0000	0.0000	0.0000	0.0000	1.0000
2	e=800	0	0.3131	0.4088	0.5019	0.5951	1.0000
3	e=70	1	1.0000	0.4498	0.9110	0.0808	0.0030
4	e=400	4	0.0001	0.0001	0.0041	1.0000	0.0000
5	e=70	1	1.0000	0.2388	0.4668	0.0516	0.0000
6	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
7	e=800	3	0.2067	0.0110	0.0005	0.0097	1.0000
8	e=70	3	1.0000	0.8014	0.0004	0.0194	0.0003
9	e=70	2	1.0000	0.7584	0.0000	0.7372	0.0000
10	e=70	3	1.0000	0.3854	0.0000	0.0014	0.0000
11	e=400	3	0.4583	0.0366	0.0003	1.0000	0.0000
12	e=800	4	0.0482	0.0085	0.0000	0.0000	1.0000
13	e=70	2	1.0000	0.9666	0.0000	0.8888	0.0000
14	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
15	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
16	e=800	3	0.0120	0.0969	0.0000	0.0036	1.0000
17	e=100	2	0.5854	1.0000	0.7058	0.0000	0.0000
18	e=70	3	1.0000	0.3199	0.0000	0.0435	0.0000
19	e=400	3	0.1290	0.0078	0.0000	1.0000	0.0000
20	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
21	e=400	2	0.2561	0.4842	0.0000	1.0000	0.0000
22	e=70	2	1.0000	0.6345	0.0000	0.1027	0.0000
23	e=70	2	1.0000	0.3931	0.0000	0.5019	0.0000
24	e=100	3	0.5382	1.0000	0.0000	0.0015	0.0000
25	e=100	3	0.6147	1.0000	0.0000	0.0000	0.0000
26	e=800	4	0.0000	0.0000	0.0000	0.0000	1.0000
27	e=800	4	0.0055	0.0023	0.0000	0.0006	1.0000
28	e=800	3	0.4009	0.0340	0.0000	0.0218	1.0000
29	e=100	2	0.4250	1.0000	0.0000	0.1733	0.0000
30	e=100	3	0.9332	1.0000	0.0000	0.0166	0.0008
31	e=100	2	0.8778	1.0000	0.0202	0.1401	0.0001
32	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
33	e=70	3	1.0000	0.0482	0.5951	0.0001	0.0000
34	e=100	3	0.4414	1.0000	0.0000	0.0353	0.0000
35	e=800	4	0.0000	0.0000	0.0000	0.0000	1.0000
36	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
37	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
38	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
39	e=200	4	0.0000	0.0000	1.0000	0.0000	0.0000
40	e=200	4	0.0000	0.0000	1.0000	0.0004	0.0000
ψ_{sp}			17	21	25	27	31

Table A.12.: Significance evaluation number of training epochs for DL-STPM.

A. Significance tables

Part	Best model	t/bm	STPM-VPD	STPM-VPD-enh	DL-STPM-VPD
1	DL-STPM-VPD	2	0.0000	0.0000	1.0000
2	STPM-VPD	2	1.0000	0.0000	0.0000
3	DL-STPM-VPD	2	0.0000	0.0000	1.0000
4	DL-STPM-VPD	2	0.0001	0.0000	1.0000
5	DL-STPM-VPD	2	0.0000	0.0000	1.0000
6	STPM-VPD	2	1.0000	0.0000	0.0000
7	STPM-VPD	2	1.0000	0.0000	0.0000
8	DL-STPM-VPD	1	0.0001	0.8339	1.0000
9	DL-STPM-VPD	2	0.0000	0.0000	1.0000
10	STPM-VPD-enh	2	0.0000	1.0000	0.0001
11	STPM-VPD-enh	2	0.0000	1.0000	0.0000
12	DL-STPM-VPD	2	0.0009	0.0041	1.0000
13	STPM-VPD-enh	2	0.0000	1.0000	0.0041
14	STPM-VPD-enh	2	0.0000	1.0000	0.0000
15	STPM-VPD	2	1.0000	0.0000	0.0000
16	STPM-VPD-enh	2	0.0000	1.0000	0.0000
17	DL-STPM-VPD	2	0.0000	0.0000	1.0000
18	STPM-VPD	2	1.0000	0.0000	0.0000
19	DL-STPM-VPD	2	0.0000	0.0000	1.0000
20	DL-STPM-VPD	2	0.0000	0.0000	1.0000
21	STPM-VPD	2	1.0000	0.0000	0.0000
22	DL-STPM-VPD	2	0.0000	0.0000	1.0000
23	STPM-VPD	2	1.0000	0.0000	0.0000
24	DL-STPM-VPD	2	0.0000	0.0000	1.0000
25	STPM-VPD	2	1.0000	0.0000	0.0000
26	DL-STPM-VPD	2	0.0000	0.0000	1.0000
27	STPM-VPD	2	1.0000	0.0000	0.0000
28	STPM-VPD-enh	2	0.0000	1.0000	0.0000
29	STPM-VPD	2	1.0000	0.0000	0.0000
30	DL-STPM-VPD	2	0.0000	0.0000	1.0000
31	STPM-VPD	2	1.0000	0.0000	0.0000
32	DL-STPM-VPD	2	0.0000	0.0000	1.0000
33	STPM-VPD	2	1.0000	0.0000	0.0009
34	DL-STPM-VPD	2	0.0000	0.0000	1.0000
35	DL-STPM-VPD	2	0.0000	0.0000	1.0000
36	STPM-VPD-enh	1	0.0000	1.0000	0.5199
37	STPM-VPD-enh	2	0.0000	1.0000	0.0000
38	DL-STPM-VPD	2	0.0000	0.0000	1.0000
39	STPM-VPD-enh	2	0.0000	1.0000	0.0000
40	STPM-VPD	2	1.0000	0.0000	0.0000
41	DL-STPM-VPD	2	0.0159	0.0000	1.0000
42	STPM-VPD-enh	2	0.0000	1.0000	0.0000
43	STPM-VPD-enh	2	0.0000	1.0000	0.0499
44	STPM-VPD	2	1.0000	0.0000	0.0499
45	STPM-VPD-enh	2	0.0000	1.0000	0.0159
46	STPM-VPD-enh	1	0.0000	1.0000	0.2805
47	STPM-VPD	2	1.0000	0.0000	0.0000
48	STPM-VPD	2	1.0000	0.0000	0.0000
49	STPM-VPD-enh	2	0.0000	1.0000	0.0000
50	DL-STPM-VPD	0	0.5199	0.2805	1.0000
51	DL-STPM-VPD	2	0.0499	0.0000	1.0000
52	STPM-VPD-enh	2	0.0000	1.0000	0.0000
53	STPM-VPD	2	1.0000	0.0000	0.0000
54	DL-STPM-VPD	1	0.5199	0.0000	1.0000
55	DL-STPM-VPD	2	0.0000	0.0000	1.0000
56	STPM-VPD-enh	1	0.0000	1.0000	0.8339
57	DL-STPM-VPD	1	0.2805	0.0000	1.0000
58	STPM-VPD	2	1.0000	0.0000	0.0000
59	DL-STPM-VPD	2	0.0001	0.0000	1.0000
60	DL-STPM-VPD	2	0.0000	0.0000	1.0000
61	DL-STPM-VPD	2	0.0000	0.0000	1.0000
62	DL-STPM-VPD	2	0.0159	0.0009	1.0000
63	STPM-VPD-enh	2	0.0000	1.0000	0.0159
64	STPM-VPD-enh	2	0.0000	1.0000	0.0499
65	DL-STPM-VPD	2	0.0000	0.0000	1.0000
66	STPM-VPD-enh	2	0.0000	1.0000	0.0000
67	STPM-VPD-enh	1	0.0000	1.0000	0.5199
68	STPM-VPD	2	1.0000	0.0000	0.0000
69	STPM-VPD-enh	2	0.0000	1.0000	0.0000
70	STPM-VPD-enh	2	0.0000	1.0000	0.0000
71	STPM-VPD	2	1.0000	0.0000	0.0000
72	STPM-VPD-enh	2	0.0000	1.0000	0.0000
73	STPM-VPD	2	1.0000	0.0000	0.0000

Table A.13.: Significance evaluation current model for DL-STPM-VPD.

Part	Best model	t/b_m	STPM-VPD	STPM-VPD-enh	DL-STPM-VPD
74	STPM-VPD	2	1.0000	0.0000	0.0000
75	DL-STPM-VPD	2	0.0000	0.0000	1.0000
76	DL-STPM-VPD	2	0.0000	0.0159	1.0000
77	STPM-VPD-enh	2	0.0000	1.0000	0.0000
78	STPM-VPD	2	1.0000	0.0000	0.0000
79	DL-STPM-VPD	2	0.0000	0.0000	1.0000
80	STPM-VPD	2	1.0000	0.0000	0.0000
81	STPM-VPD	2	1.0000	0.0000	0.0000
82	DL-STPM-VPD	1	0.0000	0.2805	1.0000
83	STPM-VPD	2	1.0000	0.0000	0.0000
84	DL-STPM-VPD	2	0.0000	0.0000	1.0000
85	DL-STPM-VPD	2	0.0000	0.0000	1.0000
86	DL-STPM-VPD	2	0.0159	0.0000	1.0000
87	DL-STPM-VPD	2	0.0000	0.0000	1.0000
88	STPM-VPD-enh	1	0.0000	1.0000	0.2805
89	DL-STPM-VPD	2	0.0000	0.0000	1.0000
90	DL-STPM-VPD	2	0.0159	0.0000	1.0000
91	DL-STPM-VPD	0	0.8339	0.5199	1.0000
92	STPM-VPD-enh	2	0.0000	1.0000	0.0000
93	DL-STPM-VPD	2	0.0159	0.0009	1.0000
94	DL-STPM-VPD	1	0.0499	0.1290	1.0000
95	DL-STPM-VPD	2	0.0000	0.0000	1.0000
96	STPM-VPD	2	1.0000	0.0000	0.0000
97	STPM-VPD-enh	2	0.0000	1.0000	0.0041
98	STPM-VPD	2	1.0000	0.0000	0.0000
99	DL-STPM-VPD	2	0.0009	0.0000	1.0000
100	STPM-VPD-enh	2	0.0000	1.0000	0.0499
101	STPM-VPD-enh	2	0.0000	1.0000	0.0000
102	STPM-VPD-enh	2	0.0000	1.0000	0.0000
103	STPM-VPD-enh	1	0.0000	1.0000	0.5199
104	STPM-VPD	2	1.0000	0.0000	0.0000
105	STPM-VPD	1	1.0000	0.0000	0.2805
106	STPM-VPD	2	1.0000	0.0000	0.0000
107	STPM-VPD	2	1.0000	0.0000	0.0000
108	STPM-VPD	2	1.0000	0.0000	0.0000
109	STPM-VPD-enh	2	0.0000	1.0000	0.0159
110	DL-STPM-VPD	2	0.0000	0.0000	1.0000
111	DL-STPM-VPD	2	0.0000	0.0000	1.0000
112	DL-STPM-VPD	2	0.0000	0.0000	1.0000
113	STPM-VPD-enh	2	0.0000	1.0000	0.0000
114	STPM-VPD	2	1.0000	0.0000	0.0001
115	DL-STPM-VPD	2	0.0000	0.0000	1.0000
116	DL-STPM-VPD	2	0.0000	0.0000	1.0000
117	STPM-VPD	2	1.0000	0.0000	0.0000
118	STPM-VPD	2	1.0000	0.0000	0.0000
119	DL-STPM-VPD	2	0.0000	0.0000	1.0000
120	DL-STPM-VPD	2	0.0000	0.0499	1.0000
121	STPM-VPD	2	1.0000	0.0000	0.0009
122	STPM-VPD-enh	2	0.0000	1.0000	0.0009
123	STPM-VPD	2	1.0000	0.0000	0.0000
124	STPM-VPD-enh	2	0.0000	1.0000	0.0000
125	DL-STPM-VPD	2	0.0499	0.0499	1.0000
126	STPM-VPD-enh	2	0.0000	1.0000	0.0000
127	STPM-VPD	2	1.0000	0.0000	0.0000
128	DL-STPM-VPD	1	0.2805	0.0159	1.0000
129	STPM-VPD-enh	2	0.0000	1.0000	0.0000
130	DL-STPM-VPD	2	0.0000	0.0000	1.0000
131	DL-STPM-VPD	2	0.0000	0.0000	1.0000
132	DL-STPM-VPD	2	0.0000	0.0041	1.0000
133	STPM-VPD-enh	2	0.0000	1.0000	0.0000
134	STPM-VPD	2	1.0000	0.0000	0.0000
135	DL-STPM-VPD	2	0.0000	0.0000	1.0000
136	STPM-VPD-enh	2	0.0000	1.0000	0.0041
137	STPM-VPD	2	1.0000	0.0000	0.0000
138	STPM-VPD-enh	2	0.0000	1.0000	0.0000
139	DL-STPM-VPD	1	0.5199	0.0000	1.0000
140	STPM-VPD	2	1.0000	0.0000	0.0000
141	DL-STPM-VPD	2	0.0000	0.0000	1.0000
142	DL-STPM-VPD	2	0.0000	0.0000	1.0000
143	STPM-VPD-enh	2	0.0000	1.0000	0.0001
144	STPM-VPD	2	1.0000	0.0000	0.0000
145	DL-STPM-VPD	2	0.0000	0.0000	1.0000
146	STPM-VPD-enh	2	0.0000	1.0000	0.0000
147	STPM-VPD-enh	2	0.0000	1.0000	0.0000

Table A.13.: Significance evaluation current model for DL-STPM-VPD cont.

A. Significance tables

Part	Best model	t/bm	STPM-VPD	STPM-VPD-enh	DL-STPM-VPD
148	DL-STPM-VPD	2	0.0000	0.0000	1.0000
149	STPM-VPD-enh	2	0.0000	1.0000	0.0000
150	DL-STPM-VPD	0	0.1290	0.1290	1.0000
151	DL-STPM-VPD	2	0.0009	0.0041	1.0000
152	STPM-VPD-enh	2	0.0000	1.0000	0.0000
153	STPM-VPD	2	1.0000	0.0000	0.0000
154	STPM-VPD-enh	2	0.0000	1.0000	0.0000
155	DL-STPM-VPD	1	0.5199	0.0041	1.0000
156	STPM-VPD	2	1.0000	0.0000	0.0000
157	STPM-VPD-enh	2	0.0000	1.0000	0.0041
158	STPM-VPD-enh	2	0.0000	1.0000	0.0000
159	STPM-VPD	2	1.0000	0.0000	0.0000
160	DL-STPM-VPD	2	0.0001	0.0000	1.0000
161	DL-STPM-VPD	2	0.0159	0.0499	1.0000
162	DL-STPM-VPD	1	0.0041	0.1290	1.0000
163	STPM-VPD	2	1.0000	0.0000	0.0000
164	STPM-VPD-enh	2	0.0000	1.0000	0.0000
165	STPM-VPD-enh	2	0.0000	1.0000	0.0001
166	DL-STPM-VPD	2	0.0000	0.0000	1.0000
167	STPM-VPD	2	1.0000	0.0000	0.0000
168	STPM-VPD	2	1.0000	0.0000	0.0000
169	STPM-VPD-enh	2	0.0000	1.0000	0.0000
170	DL-STPM-VPD	2	0.0000	0.0000	1.0000
171	DL-STPM-VPD	2	0.0000	0.0000	1.0000
172	DL-STPM-VPD	2	0.0000	0.0000	1.0000
173	STPM-VPD-enh	2	0.0000	1.0000	0.0000
174	DL-STPM-VPD	0	0.1290	0.5199	1.0000
175	DL-STPM-VPD	1	0.0000	0.5199	1.0000
176	STPM-VPD	2	1.0000	0.0000	0.0001
177	STPM-VPD-enh	2	0.0000	1.0000	0.0499
178	DL-STPM-VPD	2	0.0000	0.0000	1.0000
179	STPM-VPD-enh	2	0.0000	1.0000	0.0000
180	STPM-VPD	2	1.0000	0.0000	0.0000
181	DL-STPM-VPD	2	0.0000	0.0000	1.0000
182	DL-STPM-VPD	2	0.0000	0.0000	1.0000
183	STPM-VPD	1	1.0000	0.0000	0.5199
184	STPM-VPD-enh	2	0.0000	1.0000	0.0000
185	STPM-VPD	2	1.0000	0.0000	0.0000
186	STPM-VPD	2	1.0000	0.0000	0.0001
187	STPM-VPD-enh	2	0.0000	1.0000	0.0159
188	DL-STPM-VPD	2	0.0000	0.0000	1.0000
189	DL-STPM-VPD	1	0.0000	0.2805	1.0000
190	DL-STPM-VPD	2	0.0000	0.0000	1.0000
191	DL-STPM-VPD	2	0.0000	0.0000	1.0000
192	DL-STPM-VPD	2	0.0000	0.0000	1.0000
193	DL-STPM-VPD	2	0.0000	0.0000	1.0000
194	STPM-VPD-enh	1	0.0000	1.0000	0.1290
195	DL-STPM-VPD	1	0.0001	0.2805	1.0000
196	STPM-VPD	2	1.0000	0.0000	0.0000
197	STPM-VPD	2	1.0000	0.0000	0.0041
198	DL-STPM-VPD	2	0.0000	0.0000	1.0000
199	DL-STPM-VPD	2	0.0000	0.0000	1.0000
200	STPM-VPD-enh	2	0.0000	1.0000	0.0000
201	DL-STPM-VPD	1	0.5199	0.0041	1.0000
202	DL-STPM-VPD	2	0.0000	0.0000	1.0000
203	STPM-VPD	2	1.0000	0.0000	0.0000
204	STPM-VPD-enh	2	0.0000	1.0000	0.0000
205	STPM-VPD	2	1.0000	0.0000	0.0000
206	STPM-VPD-enh	2	0.0000	1.0000	0.0000
207	DL-STPM-VPD	2	0.0499	0.0000	1.0000
208	STPM-VPD-enh	2	0.0000	1.0000	0.0000
209	DL-STPM-VPD	1	0.0000	0.5199	1.0000
210	STPM-VPD	2	1.0000	0.0000	0.0000
211	STPM-VPD-enh	2	0.0000	1.0000	0.0000
212	DL-STPM-VPD	2	0.0000	0.0000	1.0000
213	DL-STPM-VPD	2	0.0000	0.0000	1.0000
214	DL-STPM-VPD	2	0.0000	0.0000	1.0000
215	DL-STPM-VPD	1	0.1290	0.0000	1.0000
216	STPM-VPD	2	1.0000	0.0000	0.0001
217	STPM-VPD-enh	2	0.0000	1.0000	0.0000
218	DL-STPM-VPD	2	0.0000	0.0000	1.0000
219	DL-STPM-VPD	2	0.0000	0.0000	1.0000
220	DL-STPM-VPD	2	0.0009	0.0041	1.0000

Table A.13.: Significance evaluation current model for DL-STPM-VPD cont.

Part	Best model	t/bm	STPM-VPD	STPM-VPD-enh	DL-STPM-VPD
221	DL-STPM-VPD	2	0.0000	0.0499	1.0000
222	STPM-VPD-enh	2	0.0000	1.0000	0.0000
223	DL-STPM-VPD	2	0.0000	0.0000	1.0000
224	DL-STPM-VPD	2	0.0000	0.0000	1.0000
225	DL-STPM-VPD	2	0.0000	0.0000	1.0000
226	DL-STPM-VPD	1	0.0159	0.8339	1.0000
227	STPM-VPD	2	1.0000	0.0000	0.0000
228	STPM-VPD-enh	2	0.0000	1.0000	0.0009
229	STPM-VPD	2	1.0000	0.0000	0.0000
230	STPM-VPD-enh	2	0.0000	1.0000	0.0000
231	STPM-VPD-enh	2	0.0000	1.0000	0.0000
232	STPM-VPD	2	1.0000	0.0000	0.0000
233	DL-STPM-VPD	2	0.0000	0.0000	1.0000
234	STPM-VPD-enh	2	0.0000	1.0000	0.0000
235	DL-STPM-VPD	2	0.0000	0.0000	1.0000
236	STPM-VPD-enh	2	0.0000	1.0000	0.0000
237	DL-STPM-VPD	2	0.0000	0.0000	1.0000
238	STPM-VPD	2	1.0000	0.0000	0.0000
239	STPM-VPD	2	1.0000	0.0000	0.0000
240	DL-STPM-VPD	2	0.0000	0.0000	1.0000
241	DL-STPM-VPD	2	0.0009	0.0499	1.0000
242	STPM-VPD-enh	1	0.0000	1.0000	0.8339
243	STPM-VPD	2	1.0000	0.0000	0.0009
244	DL-STPM-VPD	0	0.5199	0.1290	1.0000
245	DL-STPM-VPD	2	0.0000	0.0009	1.0000
246	DL-STPM-VPD	2	0.0041	0.0001	1.0000
247	STPM-VPD	2	1.0000	0.0000	0.0000
248	STPM-VPD	2	1.0000	0.0000	0.0000
249	STPM-VPD	2	1.0000	0.0000	0.0000
250	DL-STPM-VPD	2	0.0000	0.0001	1.0000
251	STPM-VPD	1	1.0000	0.0000	0.8339
252	DL-STPM-VPD	2	0.0000	0.0000	1.0000
253	DL-STPM-VPD	2	0.0000	0.0000	1.0000
254	DL-STPM-VPD	2	0.0000	0.0000	1.0000
255	STPM-VPD	2	1.0000	0.0000	0.0000
256	STPM-VPD-enh	2	0.0000	1.0000	0.0000
257	STPM-VPD	1	1.0000	0.0000	0.5199
258	STPM-VPD-enh	2	0.0000	1.0000	0.0000
259	DL-STPM-VPD	1	0.0159	0.2805	1.0000
260	STPM-VPD-enh	2	0.0000	1.0000	0.0000
261	DL-STPM-VPD	2	0.0000	0.0000	1.0000
262	DL-STPM-VPD	1	0.0000	0.1290	1.0000
263	STPM-VPD	2	1.0000	0.0000	0.0000
264	DL-STPM-VPD	2	0.0000	0.0041	1.0000
265	STPM-VPD	1	1.0000	0.0000	0.8339
266	DL-STPM-VPD	2	0.0000	0.0000	1.0000
267	DL-STPM-VPD	1	0.1290	0.0000	1.0000
268	DL-STPM-VPD	2	0.0499	0.0000	1.0000
269	DL-STPM-VPD	2	0.0000	0.0000	1.0000
270	DL-STPM-VPD	2	0.0000	0.0000	1.0000
271	DL-STPM-VPD	2	0.0000	0.0000	1.0000
272	STPM-VPD	2	1.0000	0.0000	0.0000
273	STPM-VPD	2	1.0000	0.0000	0.0000
274	DL-STPM-VPD	2	0.0000	0.0000	1.0000
275	DL-STPM-VPD	0	0.2805	0.1290	1.0000
276	DL-STPM-VPD	1	0.0000	0.2805	1.0000
277	STPM-VPD-enh	2	0.0000	1.0000	0.0000
278	DL-STPM-VPD	1	0.1290	0.0000	1.0000
279	DL-STPM-VPD	2	0.0000	0.0000	1.0000
280	STPM-VPD-enh	2	0.0000	1.0000	0.0000
281	DL-STPM-VPD	2	0.0000	0.0000	1.0000
282	STPM-VPD-enh	2	0.0000	1.0000	0.0000
283	STPM-VPD-enh	1	0.0000	1.0000	0.2805
284	DL-STPM-VPD	2	0.0000	0.0000	1.0000
285	STPM-VPD	2	1.0000	0.0000	0.0000
286	DL-STPM-VPD	2	0.0000	0.0000	1.0000
287	DL-STPM-VPD	2	0.0000	0.0000	1.0000
288	STPM-VPD-enh	2	0.0000	1.0000	0.0000
289	DL-STPM-VPD	2	0.0000	0.0000	1.0000
290	DL-STPM-VPD	2	0.0000	0.0000	1.0000
291	DL-STPM-VPD	2	0.0000	0.0000	1.0000
292	DL-STPM-VPD	2	0.0000	0.0000	1.0000
293	STPM-VPD	2	1.0000	0.0000	0.0000

Table A.13.: Significance evaluation current model for DL-STPM-VPD cont.

A. Significance tables

Part	Best model	ψ_{bm}	STPM-VPD	STPM-VPD-enh	DL-STPM-VPD
294	STPM-VPD	2	1.0000	0.0000	0.0000
295	STPM-VPD	2	1.0000	0.0000	0.0041
296	STPM-VPD-enh	2	0.0000	1.0000	0.0009
297	STPM-VPD-enh	2	0.0000	1.0000	0.0000
298	DL-STPM-VPD	2	0.0000	0.0041	1.0000
299	DL-STPM-VPD	2	0.0000	0.0000	1.0000
300	DL-STPM-VPD	2	0.0159	0.0000	1.0000
301	STPM-VPD-enh	2	0.0000	1.0000	0.0000
302	DL-STPM-VPD	2	0.0000	0.0000	1.0000
303	DL-STPM-VPD	2	0.0000	0.0000	1.0000
304	DL-STPM-VPD	1	0.0041	0.1290	1.0000
305	STPM-VPD	2	1.0000	0.0000	0.0001
306	STPM-VPD	2	1.0000	0.0000	0.0000
307	DL-STPM-VPD	2	0.0000	0.0000	1.0000
308	DL-STPM-VPD	1	0.0000	0.1290	1.0000
309	STPM-VPD-enh	2	0.0000	1.0000	0.0000
310	DL-STPM-VPD	2	0.0000	0.0000	1.0000
311	STPM-VPD	2	1.0000	0.0000	0.0000
312	DL-STPM-VPD	1	0.0499	0.5199	1.0000
313	STPM-VPD	2	1.0000	0.0000	0.0000
314	STPM-VPD-enh	2	0.0000	1.0000	0.0000
315	DL-STPM-VPD	2	0.0000	0.0159	1.0000
316	DL-STPM-VPD	2	0.0159	0.0499	1.0000
317	STPM-VPD	1	1.0000	0.0000	0.5199
318	DL-STPM-VPD	2	0.0000	0.0000	1.0000
319	DL-STPM-VPD	2	0.0000	0.0000	1.0000
320	STPM-VPD	2	1.0000	0.0000	0.0000
321	STPM-VPD	1	1.0000	0.0000	0.8339
322	STPM-VPD	2	1.0000	0.0000	0.0159
323	DL-STPM-VPD	2	0.0000	0.0000	1.0000
324	STPM-VPD-enh	2	0.0000	1.0000	0.0000
325	DL-STPM-VPD	2	0.0000	0.0041	1.0000
326	DL-STPM-VPD	2	0.0000	0.0000	1.0000
327	STPM-VPD-enh	2	0.0000	1.0000	0.0009
328	STPM-VPD-enh	1	0.0000	1.0000	0.8339
329	STPM-VPD-enh	2	0.0000	1.0000	0.0000
330	STPM-VPD-enh	2	0.0000	1.0000	0.0000
331	DL-STPM-VPD	2	0.0000	0.0000	1.0000
332	STPM-VPD-enh	2	0.0000	1.0000	0.0041
333	STPM-VPD-enh	2	0.0000	1.0000	0.0009
334	STPM-VPD-enh	2	0.0000	1.0000	0.0000
335	DL-STPM-VPD	2	0.0000	0.0159	1.0000
336	DL-STPM-VPD	2	0.0000	0.0000	1.0000
337	STPM-VPD	2	1.0000	0.0000	0.0000
338	STPM-VPD	2	1.0000	0.0000	0.0000
339	STPM-VPD-enh	2	0.0000	1.0000	0.0000
340	STPM-VPD-enh	2	0.0000	1.0000	0.0041
341	STPM-VPD-enh	2	0.0000	1.0000	0.0000
342	DL-STPM-VPD	2	0.0000	0.0000	1.0000
343	DL-STPM-VPD	2	0.0000	0.0000	1.0000
344	STPM-VPD	2	1.0000	0.0000	0.0000
345	STPM-VPD	2	1.0000	0.0000	0.0499
346	STPM-VPD-enh	2	0.0000	1.0000	0.0001
347	STPM-VPD	2	1.0000	0.0000	0.0000
348	STPM-VPD-enh	2	0.0000	1.0000	0.0000
349	DL-STPM-VPD	2	0.0000	0.0000	1.0000
350	STPM-VPD-enh	2	0.0000	1.0000	0.0041
351	DL-STPM-VPD	2	0.0000	0.0000	1.0000
352	STPM-VPD	2	1.0000	0.0000	0.0001
353	STPM-VPD-enh	2	0.0000	1.0000	0.0000
354	STPM-VPD-enh	1	0.0000	1.0000	0.5199
355	STPM-VPD-enh	2	0.0000	1.0000	0.0000
356	STPM-VPD-enh	1	0.0000	1.0000	0.2805
357	DL-STPM-VPD	2	0.0000	0.0000	1.0000
358	DL-STPM-VPD	2	0.0000	0.0000	1.0000
359	STPM-VPD-enh	2	0.0000	1.0000	0.0000
360	STPM-VPD	2	1.0000	0.0000	0.0041
361	STPM-VPD	2	1.0000	0.0000	0.0000
362	STPM-VPD-enh	2	0.0000	1.0000	0.0000
363	DL-STPM-VPD	2	0.0000	0.0000	1.0000
364	DL-STPM-VPD	2	0.0000	0.0000	1.0000
365	DL-STPM-VPD	2	0.0000	0.0000	1.0000
		ψ_{sp}	254	241	180

Table A.13.: Significance evaluation current model for DL-STPM-VPD cont.

Part	Best model	ψ_{bm}	STPM	STPM-enh	DL-STPM
1	DL-STPM	2	0.0000	0.0000	1.0000
2	STPM	2	1.0000	0.0000	0.0000
3	DL-STPM	2	0.0000	0.0000	1.0000
4	STPM	2	1.0000	0.0000	0.0000
5	STPM	2	1.0000	0.0000	0.0000
6	STPM-enh	2	0.0000	1.0000	0.0000
7	STPM	2	1.0000	0.0000	0.0000
8	STPM	2	1.0000	0.0000	0.0499
9	STPM	2	1.0000	0.0000	0.0000
10	DL-STPM	2	0.0000	0.0000	1.0000
11	DL-STPM	2	0.0000	0.0001	1.0000
12	DL-STPM	2	0.0000	0.0000	1.0000
13	STPM	2	1.0000	0.0000	0.0000
14	STPM	2	1.0000	0.0000	0.0000
15	STPM-enh	2	0.0000	1.0000	0.0000
16	STPM-enh	2	0.0000	1.0000	0.0000
17	STPM	2	1.0000	0.0000	0.0000
18	STPM	2	1.0000	0.0000	0.0000
19	STPM	2	1.0000	0.0000	0.0000
20	STPM	2	1.0000	0.0000	0.0499
21	STPM-enh	2	0.0000	1.0000	0.0000
22	DL-STPM	2	0.0000	0.0000	1.0000
23	STPM	2	1.0000	0.0000	0.0000
24	DL-STPM	2	0.0000	0.0000	1.0000
25	STPM-enh	2	0.0000	1.0000	0.0001
26	STPM-enh	2	0.0000	1.0000	0.0000
27	STPM-enh	2	0.0000	1.0000	0.0000
28	STPM	2	1.0000	0.0000	0.0000
29	STPM	2	1.0000	0.0000	0.0000
30	STPM	2	1.0000	0.0000	0.0000
31	STPM	2	1.0000	0.0000	0.0000
32	DL-STPM	2	0.0000	0.0000	1.0000
33	DL-STPM	2	0.0000	0.0499	1.0000
34	DL-STPM	2	0.0000	0.0000	1.0000
35	DL-STPM	2	0.0000	0.0000	1.0000
36	STPM	1	1.0000	1.0000	0.0000
37	DL-STPM	2	0.0000	0.0000	1.0000
38	DL-STPM	2	0.0000	0.0000	1.0000
39	STPM	1	1.0000	1.0000	0.0000
40	DL-STPM	1	0.0000	0.8339	1.0000
41	STPM	2	1.0000	0.0000	0.0000
42	STPM	1	1.0000	1.0000	0.0000
43	DL-STPM	2	0.0000	0.0000	1.0000
44	DL-STPM	2	0.0000	0.0000	1.0000
45	DL-STPM	2	0.0000	0.0000	1.0000
46	STPM	2	1.0000	0.0000	0.0000
47	STPM-enh	2	0.0000	1.0000	0.0159
48	STPM-enh	2	0.0000	1.0000	0.0000
49	STPM-enh	2	0.0000	1.0000	0.0000
50	STPM	2	1.0000	0.0000	0.0000
51	DL-STPM	2	0.0000	0.0000	1.0000
52	DL-STPM	2	0.0000	0.0000	1.0000
53	STPM	2	1.0000	0.0000	0.0000
54	STPM	2	1.0000	0.0000	0.0000
55	STPM-enh	1	0.0000	1.0000	0.8339
56	DL-STPM	2	0.0000	0.0000	1.0000
57	DL-STPM	2	0.0000	0.0000	1.0000
58	STPM	2	1.0000	0.0000	0.0000
59	DL-STPM	2	0.0000	0.0000	1.0000
60	DL-STPM	2	0.0000	0.0000	1.0000
61	STPM-enh	2	0.0000	1.0000	0.0000
62	STPM-enh	2	0.0000	1.0000	0.0499
63	DL-STPM	2	0.0000	0.0000	1.0000
64	STPM	1	1.0000	1.0000	0.0000
65	DL-STPM	2	0.0000	0.0000	1.0000
66	DL-STPM	2	0.0000	0.0000	1.0000
67	DL-STPM	2	0.0000	0.0000	1.0000
68	STPM	1	1.0000	1.0000	0.0000
69	DL-STPM	2	0.0000	0.0000	1.0000
70	DL-STPM	2	0.0000	0.0000	1.0000
71	STPM	2	1.0000	0.0000	0.0000
72	DL-STPM	2	0.0000	0.0000	1.0000
73	DL-STPM	2	0.0000	0.0041	1.0000

Table A.14.: Significance evaluation current model for DL-STPM.

A. Significance tables

Part	Best model	ψ_{bm}	STPM	STPM-enh	DL-STPM
74	STPM-enh	2	0.0000	1.0000	0.0000
75	DL-STPM	2	0.0000	0.0000	1.0000
76	DL-STPM	2	0.0000	0.0000	1.0000
77	STPM	2	1.0000	0.0000	0.0000
78	STPM-enh	2	0.0000	1.0000	0.0000
79	STPM	1	1.0000	1.0000	0.0000
80	DL-STPM	2	0.0000	0.0041	1.0000
81	STPM-enh	2	0.0000	1.0000	0.0000
82	STPM-enh	2	0.0000	1.0000	0.0000
83	STPM	2	1.0000	0.0000	0.0000
84	STPM-enh	2	0.0000	1.0000	0.0000
85	DL-STPM	2	0.0000	0.0000	1.0000
86	DL-STPM	2	0.0000	0.0000	1.0000
87	DL-STPM	2	0.0000	0.0000	1.0000
88	STPM	2	1.0000	0.0000	0.0000
89	DL-STPM	2	0.0000	0.0000	1.0000
90	DL-STPM	2	0.0000	0.0000	1.0000
91	STPM-enh	2	0.0000	1.0000	0.0000
92	STPM	2	1.0000	0.0000	0.0000
93	STPM-enh	2	0.0000	1.0000	0.0009
94	DL-STPM	2	0.0000	0.0000	1.0000
95	DL-STPM	2	0.0000	0.0000	1.0000
96	STPM-enh	2	0.0000	1.0000	0.0499
97	DL-STPM	2	0.0000	0.0000	1.0000
98	STPM-enh	2	0.0000	1.0000	0.0159
99	DL-STPM	2	0.0000	0.0000	1.0000
100	DL-STPM	2	0.0000	0.0000	1.0000
101	DL-STPM	2	0.0000	0.0000	1.0000
102	DL-STPM	2	0.0000	0.0000	1.0000
103	DL-STPM	2	0.0000	0.0000	1.0000
104	STPM-enh	1	0.0000	1.0000	0.2805
105	STPM	2	1.0000	0.0000	0.0000
106	DL-STPM	2	0.0000	0.0000	1.0000
107	DL-STPM	2	0.0000	0.0000	1.0000
108	DL-STPM	2	0.0000	0.0000	1.0000
109	DL-STPM	2	0.0000	0.0000	1.0000
110	DL-STPM	2	0.0000	0.0000	1.0000
111	DL-STPM	2	0.0000	0.0000	1.0000
112	DL-STPM	2	0.0000	0.0000	1.0000
113	DL-STPM	2	0.0000	0.0001	1.0000
114	DL-STPM	2	0.0000	0.0000	1.0000
115	DL-STPM	2	0.0000	0.0000	1.0000
116	DL-STPM	2	0.0000	0.0000	1.0000
117	DL-STPM	2	0.0000	0.0159	1.0000
118	STPM	1	1.0000	1.0000	0.0000
119	STPM-enh	2	0.0000	1.0000	0.0000
120	STPM-enh	2	0.0000	1.0000	0.0000
121	STPM-enh	2	0.0000	1.0000	0.0000
122	STPM	2	1.0000	0.0000	0.0000
123	STPM-enh	2	0.0000	1.0000	0.0000
124	DL-STPM	2	0.0000	0.0000	1.0000
125	STPM	2	1.0000	0.0000	0.0000
126	DL-STPM	2	0.0000	0.0000	1.0000
127	STPM	2	1.0000	0.0000	0.0000
128	STPM	1	1.0000	1.0000	0.0000
129	STPM-enh	2	0.0000	1.0000	0.0000
130	STPM-enh	2	0.0000	1.0000	0.0000
131	STPM	2	1.0000	0.0000	0.0000
132	DL-STPM	2	0.0000	0.0000	1.0000
133	STPM	1	1.0000	1.0000	0.0000
134	STPM	2	1.0000	0.0000	0.0000
135	STPM-enh	2	0.0000	1.0000	0.0159
136	DL-STPM	2	0.0000	0.0000	1.0000
137	STPM-enh	2	0.0000	1.0000	0.0000
138	DL-STPM	2	0.0000	0.0159	1.0000
139	DL-STPM	2	0.0000	0.0000	1.0000
140	DL-STPM	2	0.0000	0.0000	1.0000
141	DL-STPM	2	0.0000	0.0000	1.0000
142	STPM	2	1.0000	0.0000	0.0000
143	DL-STPM	2	0.0000	0.0000	1.0000
144	DL-STPM	2	0.0000	0.0000	1.0000
145	DL-STPM	2	0.0000	0.0000	1.0000
146	STPM-enh	2	0.0000	1.0000	0.0000
147	STPM-enh	2	0.0000	1.0000	0.0000

Table A.14.: Significance evaluation current model for DL-STPM cont.

Part	Best model	ψ_{bm}	STPM	STPM-enh	DL-STPM
148	DL-STPM	2	0.0000	0.0000	1.0000
149	STPM	2	1.0000	0.0000	0.0000
150	STPM-enh	2	0.0000	1.0000	0.0000
151	STPM-enh	2	0.0000	1.0000	0.0000
152	DL-STPM	2	0.0000	0.0000	1.0000
153	DL-STPM	2	0.0000	0.0000	1.0000
154	DL-STPM	2	0.0000	0.0000	1.0000
155	STPM	1	1.0000	1.0000	0.0000
156	STPM-enh	2	0.0000	1.0000	0.0000
157	DL-STPM	2	0.0000	0.0000	1.0000
158	DL-STPM	2	0.0000	0.0000	1.0000
159	DL-STPM	2	0.0000	0.0000	1.0000
160	DL-STPM	2	0.0000	0.0000	1.0000
161	DL-STPM	2	0.0000	0.0000	1.0000
162	STPM-enh	2	0.0000	1.0000	0.0000
163	STPM-enh	2	0.0000	1.0000	0.0000
164	STPM	1	1.0000	1.0000	0.0000
165	DL-STPM	2	0.0000	0.0000	1.0000
166	DL-STPM	2	0.0000	0.0000	1.0000
167	STPM	2	1.0000	0.0000	0.0000
168	STPM	2	1.0000	0.0000	0.0000
169	DL-STPM	2	0.0000	0.0041	1.0000
170	DL-STPM	2	0.0000	0.0000	1.0000
171	DL-STPM	2	0.0000	0.0000	1.0000
172	STPM	2	1.0000	0.0000	0.0041
173	DL-STPM	2	0.0000	0.0000	1.0000
174	DL-STPM	2	0.0000	0.0000	1.0000
175	STPM	1	1.0000	1.0000	0.0000
176	DL-STPM	2	0.0000	0.0009	1.0000
177	STPM	2	1.0000	0.0000	0.0000
178	STPM	2	1.0000	0.0000	0.0000
179	STPM	2	1.0000	0.0000	0.0000
180	STPM-enh	2	0.0000	1.0000	0.0000
181	STPM	2	1.0000	0.0000	0.0000
182	DL-STPM	2	0.0000	0.0000	1.0000
183	STPM	2	1.0000	0.0000	0.0000
184	DL-STPM	2	0.0000	0.0000	1.0000
185	STPM-enh	2	0.0000	1.0000	0.0000
186	STPM-enh	2	0.0000	1.0000	0.0000
187	STPM-enh	1	0.0000	1.0000	0.5199
188	DL-STPM	2	0.0000	0.0000	1.0000
189	DL-STPM	2	0.0000	0.0009	1.0000
190	STPM	2	1.0000	0.0000	0.0000
191	STPM	2	1.0000	0.0000	0.0000
192	DL-STPM	2	0.0000	0.0000	1.0000
193	DL-STPM	2	0.0000	0.0001	1.0000
194	DL-STPM	1	0.0000	0.8339	1.0000
195	DL-STPM	2	0.0000	0.0000	1.0000
196	STPM	1	1.0000	1.0000	0.0000
197	STPM	2	1.0000	0.0000	0.0000
198	STPM	2	1.0000	0.0000	0.0000
199	DL-STPM	2	0.0000	0.0041	1.0000
200	STPM	1	1.0000	1.0000	0.0000
201	STPM	2	1.0000	0.0000	0.0000
202	STPM	2	1.0000	0.0000	0.0000
203	DL-STPM	2	0.0000	0.0000	1.0000
204	DL-STPM	2	0.0000	0.0000	1.0000
205	STPM-enh	2	0.0000	1.0000	0.0000
206	STPM-enh	1	0.0000	1.0000	0.2805
207	STPM-enh	2	0.0000	1.0000	0.0000
208	DL-STPM	2	0.0000	0.0000	1.0000
209	DL-STPM	2	0.0000	0.0000	1.0000
210	DL-STPM	2	0.0000	0.0000	1.0000
211	DL-STPM	2	0.0000	0.0000	1.0000
212	DL-STPM	1	0.0000	0.1290	1.0000
213	STPM-enh	2	0.0000	1.0000	0.0000
214	DL-STPM	2	0.0000	0.0000	1.0000
215	DL-STPM	2	0.0000	0.0000	1.0000
216	STPM-enh	2	0.0000	1.0000	0.0000
217	STPM	2	1.0000	0.0000	0.0000
218	DL-STPM	2	0.0000	0.0000	1.0000
219	DL-STPM	2	0.0000	0.0000	1.0000
220	DL-STPM	2	0.0000	0.0000	1.0000

Table A.14.: Significance evaluation current model for DL-STPM cont.

A. Significance tables

Part	Best model	ψ_{bm}	STPM	STPM-enh	DL-STPM
221	DL-STPM	2	0.0000	0.0000	1.0000
222	STPM-enh	1	0.0000	1.0000	0.8339
223	STPM-enh	2	0.0000	1.0000	0.0000
224	DL-STPM	2	0.0000	0.0000	1.0000
225	DL-STPM	2	0.0000	0.0000	1.0000
226	STPM-enh	2	0.0000	1.0000	0.0000
227	DL-STPM	2	0.0000	0.0000	1.0000
228	DL-STPM	2	0.0000	0.0000	1.0000
229	DL-STPM	2	0.0000	0.0041	1.0000
230	STPM-enh	1	0.0000	1.0000	0.8339
231	STPM-enh	2	0.0000	1.0000	0.0000
232	DL-STPM	2	0.0000	0.0000	1.0000
233	STPM-enh	2	0.0000	1.0000	0.0000
234	DL-STPM	1	0.0000	0.2805	1.0000
235	DL-STPM	1	0.0000	0.5199	1.0000
236	STPM-enh	2	0.0000	1.0000	0.0000
237	DL-STPM	2	0.0000	0.0000	1.0000
238	DL-STPM	2	0.0000	0.0000	1.0000
239	STPM-enh	2	0.0000	1.0000	0.0159
240	DL-STPM	2	0.0000	0.0000	1.0000
241	STPM-enh	2	0.0000	1.0000	0.0000
242	STPM-enh	2	0.0000	1.0000	0.0000
243	STPM-enh	2	0.0000	1.0000	0.0000
244	STPM-enh	2	0.0000	1.0000	0.0499
245	DL-STPM	2	0.0000	0.0009	1.0000
246	STPM-enh	2	0.0000	1.0000	0.0000
247	DL-STPM	2	0.0000	0.0000	1.0000
248	DL-STPM	2	0.0000	0.0000	1.0000
249	STPM-enh	1	0.0000	1.0000	0.1290
250	STPM-enh	2	0.0000	1.0000	0.0041
251	DL-STPM	2	0.0000	0.0000	1.0000
252	STPM-enh	2	0.0000	1.0000	0.0000
253	DL-STPM	2	0.0000	0.0000	1.0000
254	STPM-enh	2	0.0000	1.0000	0.0000
255	STPM-enh	2	0.0000	1.0000	0.0159
256	STPM-enh	2	0.0000	1.0000	0.0001
257	STPM-enh	2	0.0000	1.0000	0.0000
258	DL-STPM	2	0.0000	0.0159	1.0000
259	STPM-enh	2	0.0000	1.0000	0.0000
260	DL-STPM	2	0.0000	0.0000	1.0000
261	DL-STPM	2	0.0000	0.0000	1.0000
262	STPM-enh	2	0.0000	1.0000	0.0009
263	STPM-enh	2	0.0000	1.0000	0.0000
264	DL-STPM	2	0.0000	0.0000	1.0000
265	DL-STPM	2	0.0000	0.0000	1.0000
266	STPM-enh	1	0.0000	1.0000	0.1290
267	STPM-enh	2	0.0000	1.0000	0.0000
268	STPM-enh	2	0.0000	1.0000	0.0000
269	DL-STPM	2	0.0000	0.0041	1.0000
270	STPM-enh	2	0.0000	1.0000	0.0000
271	STPM-enh	2	0.0000	1.0000	0.0000
272	STPM	2	1.0000	0.0000	0.0000
273	DL-STPM	2	0.0000	0.0000	1.0000
274	STPM-enh	2	0.0000	1.0000	0.0000
275	DL-STPM	2	0.0000	0.0000	1.0000
276	STPM-enh	2	0.0000	1.0000	0.0000
277	STPM-enh	2	0.0000	1.0000	0.0000
278	STPM-enh	2	0.0000	1.0000	0.0041
279	STPM-enh	1	0.0000	1.0000	0.8339
280	STPM-enh	2	0.0000	1.0000	0.0001
281	STPM-enh	2	0.0000	1.0000	0.0000
282	STPM-enh	2	0.0000	1.0000	0.0000
283	STPM-enh	2	0.0000	1.0000	0.0000
284	STPM-enh	1	0.0000	1.0000	0.8339
285	STPM-enh	2	0.0000	1.0000	0.0000
286	STPM-enh	2	0.0000	1.0000	0.0000
287	DL-STPM	2	0.0000	0.0000	1.0000
288	STPM	2	1.0000	0.0000	0.0000
289	DL-STPM	2	0.0000	0.0000	1.0000
290	STPM-enh	1	0.0000	1.0000	0.1290
291	STPM	1	1.0000	1.0000	0.0000
292	DL-STPM	2	0.0000	0.0000	1.0000
293	DL-STPM	1	0.0000	0.1290	1.0000

Table A.14.: Significance evaluation current model for DL-STPM cont.

Part	Best model	ψ_{bm}	STPM	STPM-enh	DL-STPM
294	DL-STPM	1	0.0000	0.2805	1.0000
295	DL-STPM	2	0.0000	0.0000	1.0000
296	DL-STPM	2	0.0000	0.0000	1.0000
297	STPM-enh	2	0.0000	1.0000	0.0000
298	DL-STPM	2	0.0000	0.0000	1.0000
299	STPM-enh	2	0.0000	1.0000	0.0000
300	DL-STPM	1	0.0000	0.2805	1.0000
301	DL-STPM	2	0.0000	0.0000	1.0000
302	STPM-enh	1	0.0000	1.0000	0.1290
303	STPM-enh	2	0.0000	1.0000	0.0000
304	DL-STPM	2	0.0000	0.0000	1.0000
305	DL-STPM	2	0.0000	0.0000	1.0000
306	DL-STPM	2	0.0000	0.0000	1.0000
307	DL-STPM	1	0.0000	0.5199	1.0000
308	DL-STPM	1	0.0000	0.1290	1.0000
309	DL-STPM	2	0.0000	0.0000	1.0000
310	STPM-enh	2	0.0000	1.0000	0.0499
311	DL-STPM	2	0.0000	0.0000	1.0000
312	DL-STPM	2	0.0000	0.0000	1.0000
313	STPM	2	1.0000	0.0000	0.0000
314	STPM	2	1.0000	0.0000	0.0000
315	DL-STPM	2	0.0000	0.0000	1.0000
316	DL-STPM	2	0.0000	0.0000	1.0000
317	DL-STPM	2	0.0000	0.0000	1.0000
318	DL-STPM	2	0.0000	0.0000	1.0000
319	DL-STPM	1	0.0000	0.1290	1.0000
320	DL-STPM	2	0.0000	0.0000	1.0000
321	STPM	1	1.0000	0.0000	0.5199
322	STPM-enh	2	0.0000	1.0000	0.0499
323	STPM-enh	2	0.0000	1.0000	0.0000
324	DL-STPM	2	0.0000	0.0000	1.0000
325	STPM-enh	2	0.0000	1.0000	0.0041
326	DL-STPM	1	0.0000	0.5199	1.0000
327	DL-STPM	1	0.0000	0.1290	1.0000
328	STPM-enh	2	0.0000	1.0000	0.0000
329	DL-STPM	2	0.0000	0.0000	1.0000
330	DL-STPM	2	0.0000	0.0000	1.0000
331	DL-STPM	2	0.0000	0.0499	1.0000
332	DL-STPM	2	0.0000	0.0000	1.0000
333	STPM-enh	2	0.0000	1.0000	0.0000
334	DL-STPM	2	0.0000	0.0000	1.0000
335	STPM-enh	2	0.0000	1.0000	0.0000
336	STPM-enh	2	0.0000	1.0000	0.0009
337	STPM-enh	2	0.0000	1.0000	0.0001
338	STPM-enh	2	0.0000	1.0000	0.0000
339	DL-STPM	2	0.0000	0.0000	1.0000
340	DL-STPM	2	0.0000	0.0000	1.0000
341	STPM-enh	2	0.0000	1.0000	0.0000
342	DL-STPM	2	0.0000	0.0000	1.0000
343	DL-STPM	2	0.0000	0.0000	1.0000
344	DL-STPM	2	0.0000	0.0159	1.0000
345	DL-STPM	2	0.0000	0.0000	1.0000
346	STPM	2	1.0000	0.0000	0.0000
347	STPM	2	1.0000	0.0000	0.0000
348	DL-STPM	2	0.0000	0.0000	1.0000
349	DL-STPM	2	0.0000	0.0499	1.0000
350	DL-STPM	2	0.0000	0.0000	1.0000
351	STPM-enh	2	0.0000	1.0000	0.0499
352	DL-STPM	2	0.0000	0.0000	1.0000
353	DL-STPM	2	0.0000	0.0000	1.0000
354	STPM-enh	2	0.0000	1.0000	0.0000
355	STPM-enh	1	0.0000	1.0000	0.5199
356	DL-STPM	2	0.0000	0.0000	1.0000
357	STPM-enh	2	0.0000	1.0000	0.0000
358	STPM	1	1.0000	1.0000	0.0000
359	DL-STPM	2	0.0000	0.0000	1.0000
360	DL-STPM	2	0.0000	0.0000	1.0000
361	STPM-enh	2	0.0000	1.0000	0.0000
362	DL-STPM	2	0.0000	0.0000	1.0000
363	DL-STPM	2	0.0000	0.0000	1.0000
364	DL-STPM	2	0.0000	0.0000	1.0000
365	DL-STPM	2	0.0000	0.0000	1.0000
		ψ_{sp}	291	228	168

Table A.14.: Significance evaluation current model for DL-STPM cont.

Declaration of Authorship

I hereby declare that this thesis was created by me and me alone using only the stated sources and tools.

Robby Henkelmann

Magdeburg, June 14, 2018