Bayes Networks Winter 2017/2018
Prof. Dr. R. Kruse / Alexander Dockhorn

Exercise Sheet 10

Markov Properties of Undirected Graphs

Let $(\cdot \perp\!\!\!\perp \cdot \mid \cdot)$ be the ternary relation that represents the conditional independence statements that hold true in a probability distribution p over a common domain and set V of attributes. An undirected graph $G = (V, E)$ satisfies the

pairwise Markov property
if and only if every pair of non-adjacent attributes in the graph are conditional independent in p given all other attributes,i. e.

$$\forall A, B \in V, A \neq B : (A, B) \notin E \Rightarrow A \perp\!\!\!\perp B \mid V \backslash \{A, B\}.$$

G has the **local Markov property**
if and only if every attribute in p is conditionally independent of all others given its neighbors,i. e.

$$\forall A \in V : A \perp\!\!\!\perp V \backslash \{A\} \backslash \text{neighbors}(A) \mid \text{neighbors}(A),$$

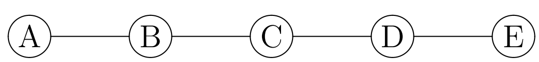
with $\text{neighbors}(A) = \{B \in V \mid (A, B) \in E\}$,

G has the **global Markov property**
if and only if from u-separation of two sets of attributes given a third one it follows that these two sets are conditionally independent in p given the third one, i. e.

$$\forall X, Y, Z \subseteq V : \langle X \mid Z \mid Y \rangle_G \Rightarrow X \perp\!\!\!\perp Y \mid Z.$$

Exercise 31 Markov Properties of Undirected Graphs

Consider the following graph:



Let $\text{dom}(A) = \cdots = \text{dom}(E) = \{0, 1\}$. Assuming the probability distribution $P(A = 0) = P(E = 0) = \frac{1}{2}$, $A = B$ (i. e. $P(B = 0 \mid A = 0) = 1$ and $P(B = 1 \mid A = 1) = 1$), $D = E$ and $C = B \cdot D$, show that the graph satisfies the pairwise and local but not the global Markov property.

Exercise 32 Dempster-Shafer Theory

Specify for all following mass distributions over $\Omega = \{1, 2, 3\}$ the respective belief and plausibility function (missing table entries denote 0).

	m_1	m_2	m_3	m_4	m_5
$\emptyset$					
$\{1\}$			0.2		0.25
$\{2\}$		1	0.5	0.4	
$\{3\}$			0.3		
$\{1,2\}$				0.1	
$\{1,3\}$					
$\{2,3\}$				0.5	0.75
$\{1,2,3\}$	1				

Exercise 33 Dempster-Shafer Theory

Homicide was committed. The circle of suspects consists of three persons:

$$\Omega = \{\mathsf{Antony}, \mathsf{Beth}, \mathsf{Charly}\}$$

We assume that exactly one of these persons has committed the homicide. Two witnesses provide us with the following evidence:

- $m_1(\{\mathsf{Antony}, \mathsf{Beth}\}) = 0.8$ und $m_1(\{\mathsf{Charly}\}) = 0.2$
- $m_2(\{\mathsf{Antony}, \mathsf{Charly}\}) = 0.3$ und $m_2(\{\mathsf{Beth}\}) = 0.7$

Calculate $m_1 \oplus m_2$ and $\mathrm{Bel}_1 \oplus \mathrm{Bel}_2$ for the arguments $\emptyset$,$\{\mathsf{Antony}\}$,$\{\mathsf{Beth}\}$ und $\{\mathsf{Charly}\}$.