## Assignment Sheet 3

## Assignment 9 Characteristic Function

What is the characteristic function of the set  $E = \{x \mid x \neq x\}$ ?

## Assignment 10 $\alpha$ -cuts

Compute the sets of  $\alpha$ -cuts for both

a) the two fuzzy sets  $\mu_1$  and  $\mu_2$  given by their graphs as follows



and

b) the fuzzy set defined as follows

$$\mu(x) = \begin{cases} 1 - (x - 2)^2, & \text{if } 1 \le x \le 3\\ 0, & \text{otherwise.} \end{cases}$$

## Assignment 11 Representation of Fuzzy Sets

Let  $(A_{\alpha})_{\alpha \in [0,1]}$  be the system of sets defined by

$$A_{\alpha} = \begin{cases} \left[1 - \sqrt{\ln \frac{1}{\alpha}}, 1 + \sqrt{\ln \frac{1}{\alpha}}\right], & \text{if } \alpha > 0\\ \text{IR}, & \text{if } \alpha = 0. \end{cases}$$

- a) Show that this system of sets satisfies the conditions that are satisfied by the set of  $\alpha$ -cuts of a fuzzy set (as stated in a theorem of the lecture), *i.e.* 
  - (i)  $[\mu]_0 = U$ , where  $U = \mathbb{R}$  in this case,

(ii) 
$$\forall \alpha, \beta : \quad \alpha \leq \beta \Rightarrow [\mu]_{\alpha} \supseteq [\mu]_{\beta},$$

 $\text{(iii)} \ \forall \beta \in [0,1]: \quad \bigcap_{\alpha:\alpha < \beta} [\mu]_{\alpha} = [\mu]_{\beta}.$ 

b) Find the membership function  $\mu$  of the fuzzy set that corresponds to this system of sets.